U3A Maths

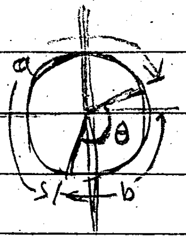
18-Mar-2025 Term 1 / Week 7

Square Root

Review - Fibonacci & Golden Ratio

Golden Angle

$$= \frac{a}{b} = \frac{(a+b)}{a} = \varphi \simeq 1.618$$

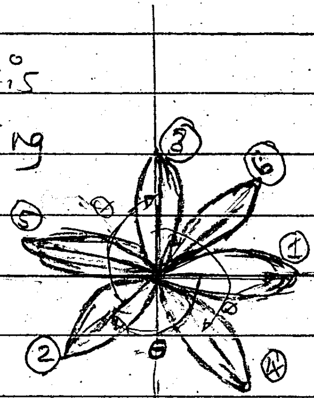


$\therefore \theta = 137.5°$

This angle commonly occurs in evolution of plant leaves to maximise the sun falling on them when new leaves form

-2-

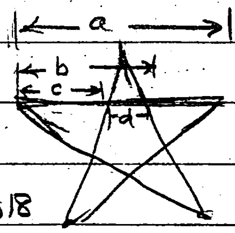
Note that 137.5° avoids overlapping of leaves!



Try with 90°, 120° or 180° and check the results!

- Pentagram

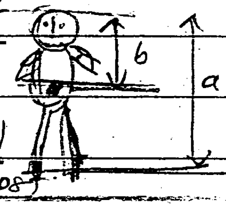
$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \varphi \simeq 1.618$$



- Divine proportion in Art

$$\frac{a}{b} = \varphi \simeq 1.618$$

b - dist to belly button!
Vitruvian Man!! (0.608)



-3-

Square Root

Division Method

$\sqrt{4} = 2$: 2 | 4; −4; 0

$\sqrt{144} = 12$: 1 | 144; −1; 22 | 044; −044; 000

$\sqrt{1024} = 32$: 3 | 1024; −9; 62 | 124; 124; 000

$\sqrt{2} = 1.414\ldots$

1	2.00 00 00
1	−1
24	100
4	−96
281	400
1	−281
2824	11900
4	11296
2828	604

-4-

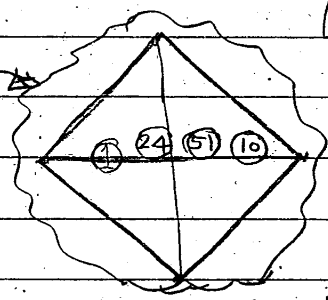
Babylonian Tablet

YouTube: Channel - Mind Your Decisions

Topic: Ancient trick to calculate Any Square Root

Babylonian
1800-1600 BCE

Stone Tablet



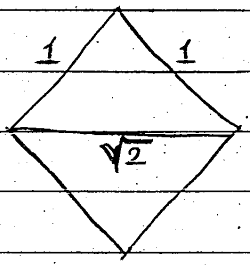
- The above numbers (in base 60) were carved on a tablet!
- Based on base 60, the numbers were interpreted as

$$1 + \frac{24}{60} + \frac{51}{60^2} + \frac{10}{60^3}$$

$$= 1.41421\ldots$$

-5-

- The value represented the length of the diagonal!



- Babylonians also provided a method to calculate square roots!

- First let us list the table of squares

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
1	4	9	16	25	36	49	64	81	100

-6-

Ex.1.

- Let us calculate $\sqrt{17}$

Refering to the table

$\sqrt{17} \simeq 4$

ie, between 4^2 & 5^2 and closer to 4^2

Continuing with the method

Given value ↓ ↓ Assumed Root,

$$\sqrt{17} \simeq 4 + \frac{(17-4^2)}{2\times 4}$$

$$= 4 + \frac{1}{8} = \underline{\underline{4.125}}$$

Exact Value: $\sqrt{17} = \underline{\underline{4.123}}$!

-7-

Ex.2

$$\sqrt{69} \simeq 8 + \frac{(69-8^2)}{2\times 8} \simeq \underline{\underline{8.3125}}$$

Exact value: $\sqrt{69} = \underline{8.307}$

Ex.3 $\sqrt{23} \simeq 5 + \frac{(23-5^2)}{2\times 5} = \underline{\underline{4.8}}$

Closer to 25 Exact Value = $\underline{\underline{4.796}}$!

Ex.4 $\sqrt{2} \simeq 1 + \frac{(2-1^2)}{2\times 1} = 1 + \frac{1}{2} = \underline{\underline{1.5}}$

We can improve this using the same technique further

$$\sqrt{2} \simeq 1.5 + \frac{(2-1.5^2)}{2\times 1.5} = \underline{\underline{1.417}}$$

We can continue the process for more accurate values!

-8-

Proof!

Let us find, say, $\sqrt{S}$

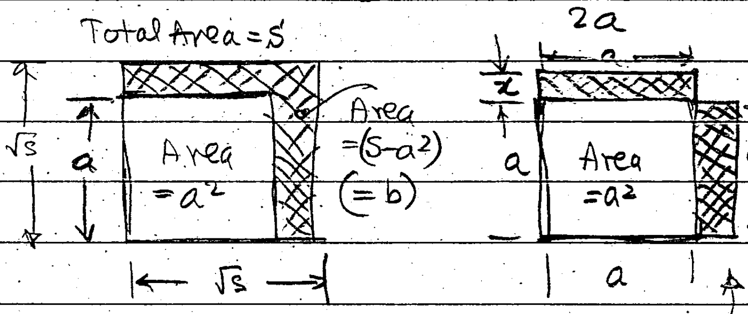
Let first approximation be $\sqrt{S} \simeq a$

$\therefore S \simeq a^2$

Let Error $b = S - a^2$

Babylonian method

$$S \simeq a + \frac{b}{2a}$$



- Redistributing the Area $(S-a^2)$

$2 \times (x \times a) = b \quad \therefore x = \underline{\underline{\frac{b}{2a}}}$

$\therefore$ We have

$$\sqrt{S} \simeq \left(a + \frac{b}{2a}\right) \;!!$$