U3A Maths

29-Mar-2025 Term 2 / Week 1

Square Root - Babylonian Method
(1800-1600 BC)

Babylonian Method

Let us calculate, say, $\sqrt{S}$

Let 'a' be the first approximation

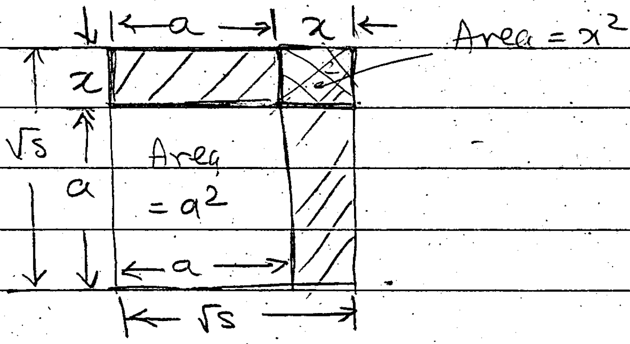
$\therefore \sqrt{S} \simeq a$

$\therefore$ Error $= (S - a^2)$

As per Babylonian method

$$\boxed{\sqrt{S} \simeq a + \frac{(S-a^2)}{2a}}$$

An elegant proof can be obtained by



Total area $= \sqrt{S} \times \sqrt{S} = S$

Also $S = a^2 + 2(x \times a) + x^2$

If the estimate 'a' is close to '$\sqrt{S}$', then the

value of 'x' will be small compared with 'a'.
Hence, area 'x^2' will be very small (negligible) compared to the total area, i.e., $\sqrt{S} \times \sqrt{S} = S$

$\therefore$ We can neglect x^2!

We have: $S \simeq a^2 + 2xa$

$\therefore$ Estimate for $x \simeq \frac{(S-a^2)}{2a}$

$\therefore$ The square root

$$\sqrt{S} \simeq a + x$$
$$\simeq a + \frac{(S-a^2)}{2a}$$

- The beauty of the method is that above process can be repeated to get better estimates of the square root.
- The initial estimate is obtained by comparing the value of 'S' in a table of squares!
- This estimate results in surprisingly accurate results with one iteration!

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Ex.1 Calculate $\sqrt{150}$.

- We have $S = 150$
- The first estimate is obtained by a table of squares

- Let first estimate be $a = 12$, Since $12^2 = 144$ is closer to 150

$12^2 = 144$
$13^2 = 169$

$$\therefore \sqrt{S} \simeq a + \frac{(S-a^2)}{2a}$$

$$= 12 + \frac{(150-144)}{2\times 12}$$

$$= 12 + \frac{6}{2\times 12} = \underline{12.25}$$

As per calculator: $\underline{12.2474\ldots}$!!

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Ex.2 Calculate $\sqrt{156}$

We have $S = 156$

This value is midway between 144 & 169

$12^2 = 144$
$13^2 = 169$

Let us try $a = 12$ again

$$\sqrt{S} = 12 + \frac{(156-12^2)}{2\times 12}$$

$$= 12 + \frac{12}{2\times 12} = \underline{12.5}\ !$$

As per calculator $\sqrt{156} = 12.48999\ldots$

- using next iteration $a = 12.5$

$$\sqrt{S} \simeq 12.5 + \frac{156-(12.5)^2}{2\times 12.5}$$

$$= 12.5 - 0.01$$

$$= 12.49\ !!$$

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Home work

- Calculate $\sqrt{156}$ using estimate $a = 13$.
Perform 2 iterations.

Table of Squares

- One of the requirements of the above method is the availability of "Table of Squares". It may become tedious for larger values.
- A clever method is given below:

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- Let us say we know $13^2 = 169$ i.e. say $x^2 = 169$ where $x = 13$.

$10^2 = 100$
$11^2 = 121$
$12^2 = 144$
$13^2 = 169$
$14^2 = ??$

- We now need to calculate 14^2 or $(x+1)^2$
- We have $(x+1)^2 = x^2 + 2x + 1$
- $14^2 = 13^2 + 2\times 13 + 1$

$= 169 + 27 = 196$

- Continuing, we have

$14^2 = 196$

$15^2 = 196 + 2\times 14 + 1 = 225$

$16^2 = 225 + 2\times 15 + 1 = 256$

$17^2 = 256 + 2\times 16 + 1 = 289$ etc