U3A Maths {Week 2 & 3 No class}

20-May-2025 Term 2 / Week 4

Algebra - Basics

Variable

- In week 2, we said that colloquially an algebraic variable is used to represent an object

For Ex: 4 apples ⇒ 4 a
It can also be 4 b!

- The above definition was refined by Rene Des Cartes (1596 - 1650) as below:

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Variable ⇒ unknown value (x, y, z)
Constants ⇒ Known values (a, b, c)

- The above convention is still used where feasible, but not strictly.

- Strictly, a variable is a letter (symbol) which can take a series of known values

For Ex: $x = a$
where a can be any number

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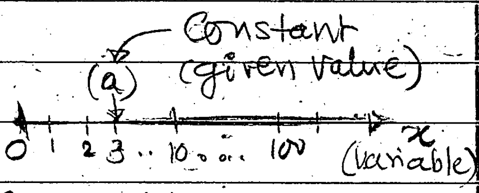
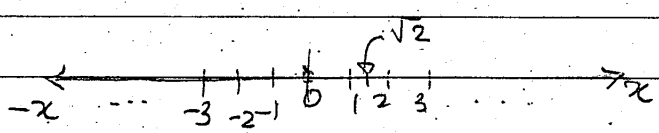
- The above can be represented graphically (geometrically) as below:



Of course, 'x' can also take negative values!



- Can we represent $\sqrt{-2}$ on the above graph!

- $\sqrt{-2}$ is an undefined value! but can be written as $(\sqrt{2})(\sqrt{-1})$

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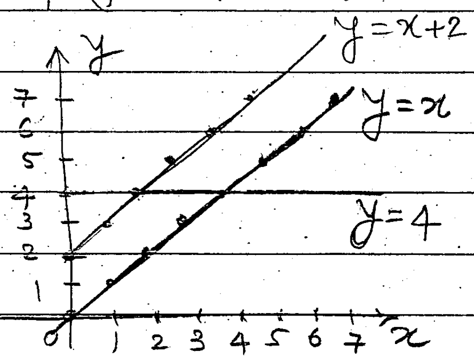
- We can also have 2 variables in a given equation!

Ex(1): $y = 4$ (for all values of 'x')!!



Ex(2): $y = x$
If $x = 1$ then $y = 1$
$x = 2$ then $y = 2$ etc

Ex(3)
Ex:
$y = x + 2$
If $x = 1$, then $y = 1+2 = 3$
Ex: 3 $x = 2$ then $y = 2+2 = 4$
$y = 4$ $x = 3$ then $y = 3+2 = 5$ etc

- Variables are also useful for developing equations for geometrical figures and other physical relationships.

Ex: Area of a rectangle

$$R = l \times b$$

Area of a circle:

$$C = \pi \times r^2$$

velocity of an object

$$v = d/t$$

- Coming back to $\sqrt{-1}$, even though we don't know the value, we can still

represent it by a symbol. (variable?)

$i = \sqrt{-1}$ (imaginary number!)

we don't know the value of 'i' but we do know that $i^2 = -1$.

Ex: Calculate $\sqrt{i}$

(Note: Basic concepts in complex numbers is assumed)

Assuming the result to be a complex number,

let $(a + ib) = \sqrt{i}$

Find a & b?

Squaring: $(a + ib)^2 = (\sqrt{i})^2 = i$

$$(a^2 + 2iab + i^2b^2) = (0 + 1i)$$

$$(a^2 - b^2) + i(2ab) = (0 + 1i)$$

Equating Real & 'i' terms

$$(a^2 - b^2) = 0 \quad \text{or} \quad a^2 = b^2$$

$$(2ab) = 1 \quad \text{or} \quad b = 1/2a$$

Substituting

$$a^2 = (1/2a)^2 = \frac{1}{4a^2}$$

$$\text{i.e.}, (a^2)^2 = \frac{1}{4} \quad \therefore a^2 = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

$$\therefore a = \frac{1}{\sqrt{2}} \quad \& \quad b = 1/\sqrt{2}$$

(Note: Ignoring negative roots)

$$\therefore \sqrt{i} = \left(1/\sqrt{2} + i\, 1/\sqrt{2}\right)$$

H.W.

You can check it by squaring!

Alternative Method

Using Euler's form for complex numbers

$$i = (0 + 1i) \Rightarrow 1e^{i90^\circ}$$

$$= 1(\cos 90^\circ + i \sin 90^\circ)$$

$$= (0 + 1i)$$

$$\therefore \sqrt{i} = \left(1e^{i90^\circ}\right)^{1/2} = 1^{1/2} \cdot e^{i45^\circ}$$

$$= 1(\cos 45^\circ + i \sin 45^\circ)$$

$$= \left(1/\sqrt{2} + i\, 1/\sqrt{2}\right)$$

{Alternative perspective in life is always useful!}

Homework

Calculate $\frac{x^2 - 16}{x - 4}$ for $x = 4$