U3A Maths

27-May-2025 Term 2/Week5

Trigonometric Functions

- Trigonometric functions are an essential part of Algebra. It was effectively integrated into Algebra by Euler's equation:

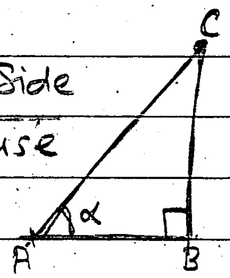
$$e^{i\theta} = (\cos\theta + i\sin\theta)$$

- Interestingly, the above equation brings together algebra, trigonometric functions and complex numbers!

- Let us first have a quick introduction to Sine (sin), complementary sine or 'cosine' (cos) and tangent (tan) functions

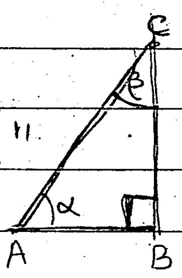
- The above functions can be defined in a simple way by using a right-angled triangle.

$$\sin(\alpha) = \frac{\text{opposite Side}}{\text{Hypotynuse}} = \frac{BC}{AC}$$



Also; $\cos(\alpha) = \dfrac{\text{Adjacent Side}}{\text{Hypotynuse}} = \dfrac{AB}{AC}$

- The angle 'β' is defined as the "complementary angle" to the angle 'α'



- We have,

$$\sin(\beta) = \frac{\text{opp. Side}}{\text{Hypot.}} = \frac{AB}{AC}$$

(Note also that $\beta = 90^\circ - \alpha$) $= \cos(\alpha)$

- Hence, Cosine (α) is the 'sine' of complementary angle 'β' !!

Lastly,

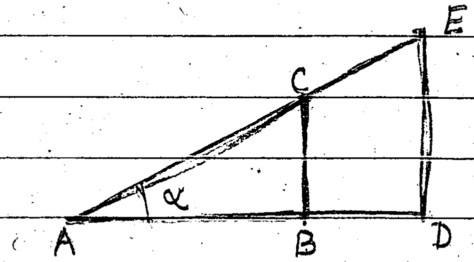
$$\tan(\alpha) = \frac{\text{opp. Side}}{\text{Adj Side}} = \frac{BC}{AB} = \frac{(BC)/(AC)}{(AB/AC)} = \frac{\sin(\alpha)}{\cos(\alpha)}$$

Note:

- A more comprehensive definitions of trigonometric functions are derived by considering the chord of a circle.

- This will be done in the next class, along with the origin of the 'sine' function.

- It can be noted that 'sine' value for a given angle is independent of the size of the triangle!



We have,

$\Delta ABC \Rightarrow \sin(\alpha) = \frac{BC}{AC}$

$\Delta ADE \Rightarrow \sin(\alpha) = \frac{DE}{AE}$

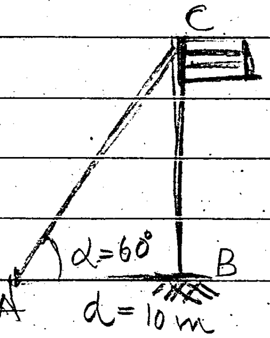
Using proportionate identity:

$$\frac{BC}{DE} = \frac{AC}{AE}$$

- The same is true for $\cos(\alpha)$ and $\tan(\alpha)$.

- Hence, a table of sin, cos and tan for various values of 'α' is very useful for solving practical problems.

Ex.1) Find the height of the flag pole, given the measured angle ($\alpha = 60°$) at a distance ($d = 10$ m) from the pole.



We have

$\tan(60°) = \frac{BC}{AB}$

$\therefore BC = AB \times \tan(60°)$

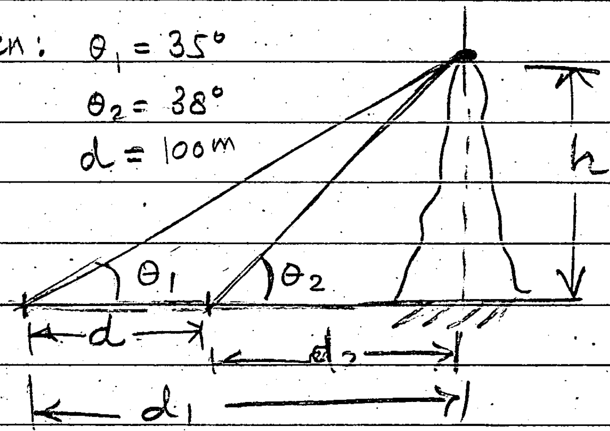
$= 10 \times 1.732 = \underline{17.32 \text{ m}}$

Ex.1(a) (Home work)

Calculate the height of the flag pole, if the angle measuring device is mounted on 2 m high stand (pole) located at 'A'. The angle measured is given to be 70°.

Ex.2 Calculate the height of the hill shown, based on the measurements below:

Given: $\theta_1 = 35°$
$\theta_2 = 38°$
$d = 100$ m



We have:

$\tan(\theta_1) = \frac{h}{d_1}$ $\quad \tan(\theta_2) = \frac{h}{d_2}$

$\therefore d_1 = h/\tan(\theta_1)$ $\quad d_2 = h/\tan(\theta_2)$

$(d_1 - d_2) = 100 = \frac{h}{\tan(35°)} - \frac{h}{\tan(38°)}$

$\therefore h = \underline{674.76 \text{ m}}$