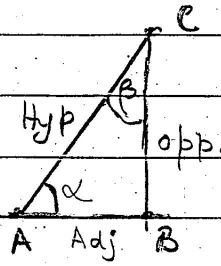
U3A Maths

3-Jun-2025 Term 2/Week 6

Trigonometric Functions - Cont'd

- Last week we defined the trigonometric functions using a right angled triangle.

- Considering the angle 'α'



Sine

$$\sin(\alpha) = \frac{opp}{Hyp}$$

Cosine

$$\cos(\alpha) = \frac{Adj}{Hyp}$$

{Same as 'Sine' of the complementary angle 'β' or $\sin(\beta)$}

Tangent

$$\tan(\alpha) = \frac{opp}{Adj} = \frac{\sin(\alpha)}{\cos(\alpha)}$$

- The reciprocals of the above functions are:

cosecant

$$\mathrm{Cosec}(\alpha) = \frac{1}{\sin(\alpha)} = Hyp/opp$$

Secant

$$\sec(\alpha) = \frac{1}{\cos(\alpha)} = Hyp/Adj$$

Cotangent

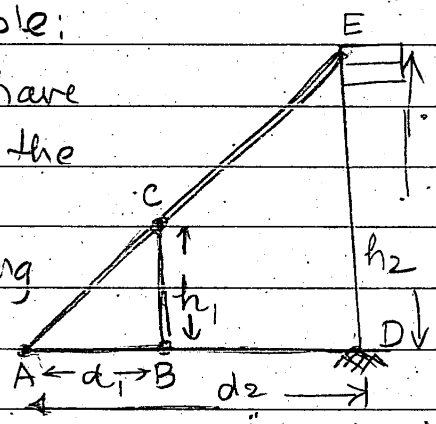
$$\cot(\alpha) = \frac{1}{\tan(\alpha)} = Adj/opp$$

- For simplicity, we used a right angled triangle, for the above definitions.
- However, we could have solved the "examples",

without defining 'Sin', 'Cos' & 'Tan' functions!

- For Example:

We could have calculated the flag pole height, using a pole of height 'h_1' at a distance 'd_1' from "eye level"!



- Using similar triangles, we have

$$\frac{h_2}{d_2} = \frac{h_1}{d_1}$$

$\therefore$ Height of the pole

$$h_2 = \left(\frac{h_1}{d_1}\right) d_2$$

- The above method (theorems on ratios of triangles) were known to Egyptians (≃ 3000 BC) & Babylonians (≃ 2000 BC)
- Usage of "chords" (sine) was introduced by Greek & Hellenistic (≃ 500 BC to 200 BC) mathematicians and were later refined by Indian mathematicians (≃ 400 AD - 600 AD).

- The term 'sine' was formally introduced by Indian mathematecians (≃ 400 AD)
- They used the term "jya" meaning "bowstring" or "chord". They also used the term "Kojya" for "cosine"!

- The "Islamic" mathematecians (≅ 800 AD) translated "jya" as "jiba". This was later (mistakenly ?) translated as "sinus" (using the term "jayb") which translated to "inlet" or "bay".

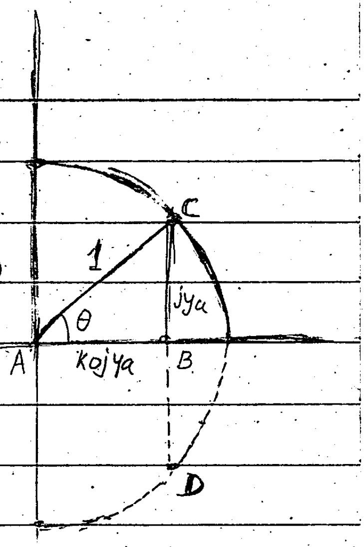
- "Sine" was essentially defined to obtain the length of a "chord" from an "arc" of circle.

- This was used for making astronomical measurements.

- For a unit circle:

$\sin(\theta) = BC$ (jya)

$\cos(\theta) = AB$ (kojya)



- Note that the length 'AB' is half the "chord length"!

- The Indian astronomers divided the 90° arc into 24 sections and calculated the value of "sines" for every 3.75° ($90/24 = 3.75$)

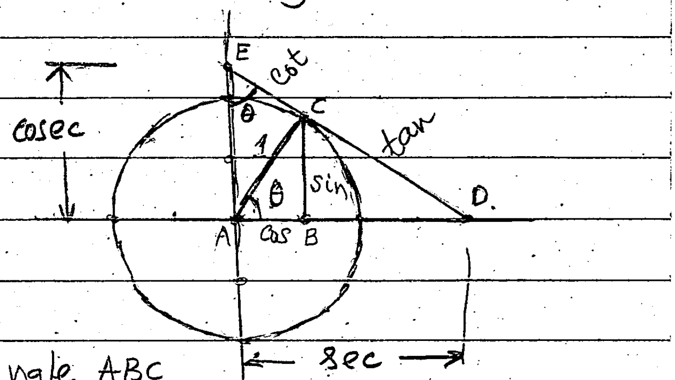
- Note: Ptolemy had developed the length of chords (≅ 150 AD) at every ½ degree, for his astronomical calculations — which was essentially a 'sine' ~~table~~.

- How do we define a tangent?

- The tangent (tan) function was developed to measure the length of the shadow cast on a sundial.

- The part which casts a shadow on a sundial is called a "gnomon"

- We are now in a position to define all the trigonometric functions, using a unit circle!



Triangle ABC

$\sin(\theta) = BC/1 \qquad \cos(\theta) = AB/1$

Triangle ACD

$\tan\theta = CD/1$

Triangle ACE

$\cot(\theta) = CE/1$

Triangle ACD $\quad \sec(\theta) = AD/1$

Triangle ACE $\quad \mathrm{cosec}(\theta) = AE/1$