U3A Maths

10-Jun-2025 Term 2/Week 7

Trigonometry - Cont'd

Review

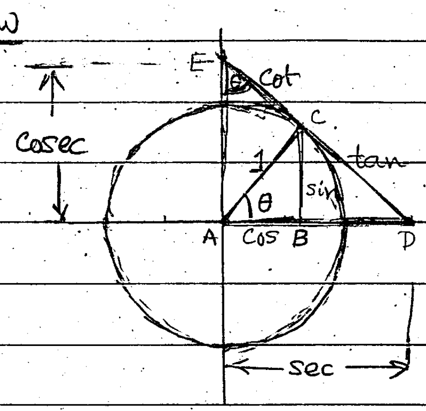

Fig. 1

ΔABC : $\sin(\theta) = \frac{OPP}{HYP} = \frac{BC}{1}$

$\cos(\theta) = \frac{ADJ}{HYP} = \frac{AB}{1}$

ΔACD : $\tan(\theta) = \frac{OPP}{ADJ} = \frac{CD}{1}$

ΔACE : $\cot(\theta) = \frac{ADJ}{OPP} = \frac{CE}{1} = \frac{1}{\tan(\theta)}$

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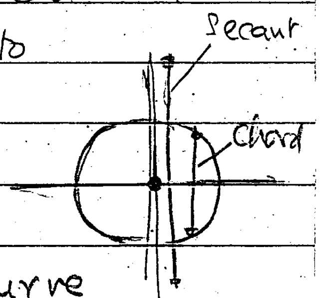
ΔACD : $\sec(\theta) = \frac{AD}{1} = \frac{HYP}{ADJ} = \frac{1}{\cos(\theta)}$

ΔACE : $\text{cosec}(\theta) = \frac{AE}{1} = \frac{HYP}{OPP} = \frac{1}{\sin(\theta)}$

Secant

- The term 'Secant' is derived from the latin word "secre" (to cut or sectionalise)
- A secant line "cuts" a given circle into two parts.



- In general, secant is a line that intersects a curve at a minimum of 2 distinct points.

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Re-writing our diagram in Fig 1

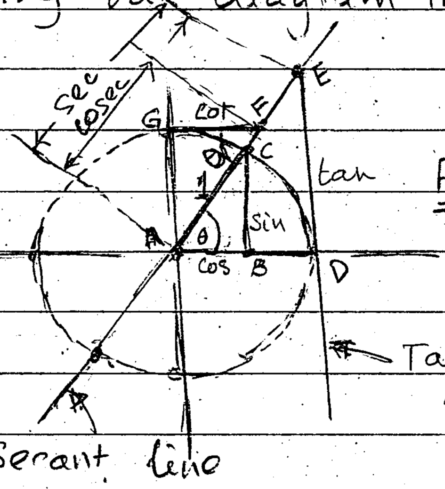

Fig. 2.

we have,

ΔADE : $\sec(\theta) = \frac{HYP}{ADJ} = \frac{AE}{AD} = \frac{AE}{1}$

ΔAGE : $\text{cosec}(\theta) = \frac{HYP}{OPP} = \frac{AF}{AG} = \frac{AF}{1}$

Note: "Secant" value is where tangent line cuts the secant line!

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- The main motivation for the trigonometric functions was astronomical measurement.
- Greek astronomers had introduced a model for the universe with the stars as a gaint sphere!
- Abu al-Wafa (Persian) (940-998 AD) was the first astronomer to develop methods of measuring distance between stars using trigonometric functions. (Book - Almagest)

Note:

In the early days "versin(θ)" function was also popular

In Fig 2, $\text{versin}(\theta) = \frac{BD}{1} = [1 - \cos(\theta)]$

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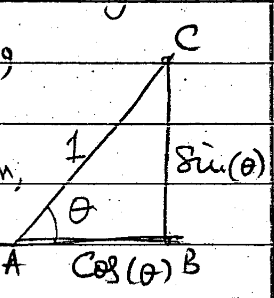
- Considering the triangle ABC in Fig 1,



Using Pythagoras theorem,

$\sin^2(\theta) + \cos^2(\theta) = 1^2$

that is, $\boxed{\sin^2(\theta) + \cos^2(\theta) = 1}$

or $\cos^2(\theta) = 1 - \sin^2(\theta)$

- Another useful trigonometric equation is the "Law of Sines", which was originally used by Cladius Ptolemy! (100-170 AD)!!

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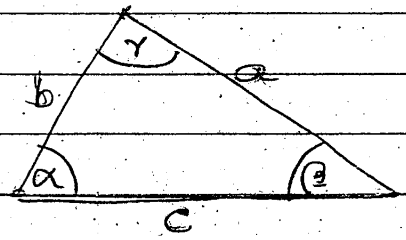
- For a any given triangle

Proof?

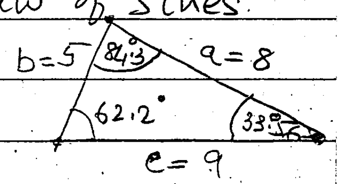


Law of Sines

$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

Note: Angles are opposite to the corresponding sides.

Ex: 1 check law of sines.



$\frac{8}{\sin(62.2)} = 9.044\ldots$

$\frac{5}{\sin(33°56)} = 9.045$

$\frac{9}{\sin(84°.3)} = 9.045$

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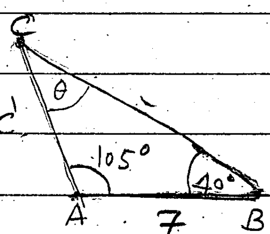
Ex. 2 Calculate the lengths 'AC' & 'BC'



we have $\theta = 180 - 105° - 40°$
$= 35°$

$\therefore \frac{AC}{\sin(40°)} = \frac{7}{\sin(35°)}$ $\therefore AC = \frac{7\sin(40°)}{\sin(35°)}$

$= 7.85$

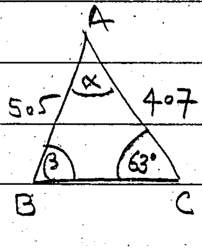
& $\frac{BC}{\sin(105°)} = \frac{7}{\sin(35°)}$ $\therefore BC = \frac{7\sin(105°)}{\sin(35°)}$

$= 11.79$

Note: This is very useful for land surveying!

Ex: 3

Calculate angle "β"



Ans: $\beta = 49°.6$

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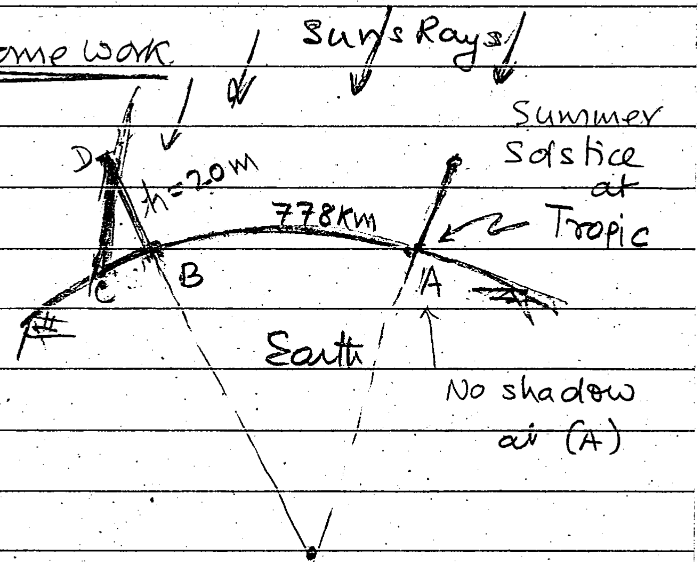
Home work



- At 12 Noon of summer solstice at Point A, Sun is directly overhead
- At point 'B', a pole of height of h = 20m casts a shadow (BC) of length 2.456 m.
- Point 'B' is at distance of 778 km from point 'A' on the same latitude.
- Calculate the Earth's circumference!