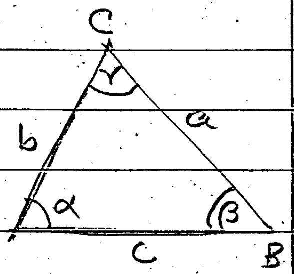
U3A Maths

17 - Jun - 2025 Term 2 / Week 8

Law of Sines - Triangulation

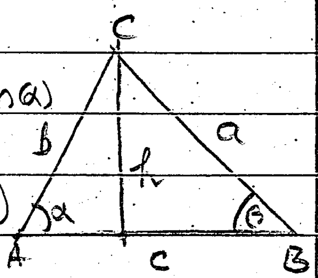
Review

Law of Sines



$$\frac{a}{\sin(\alpha)} = \frac{b}{\sin(\beta)} = \frac{c}{\sin(\gamma)}$$

Proof:



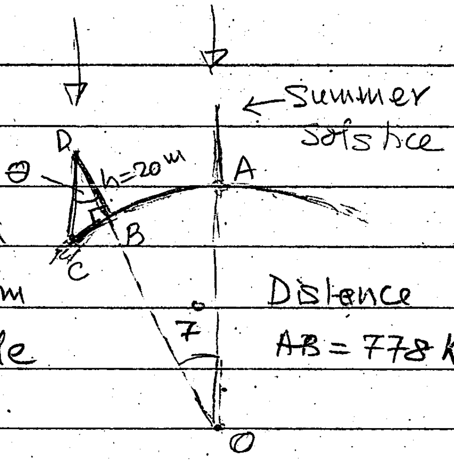
$\sin(\alpha) = \frac{h}{b}$ $\therefore h = b\sin(\alpha)$

$\sin(\beta) = \frac{h}{a}$ $\therefore h = a\sin(\beta)$

$\therefore b\sin(\alpha) = a\sin(\beta)$

$\therefore a/\sin(\alpha) = b/\sin(\beta)$

Homework



Shadow length $BC = 2.456$ m

height of pole $DB = 20$ m.

Distance $AB = 778$ km

$\therefore \tan(\theta) = \frac{BC}{DB} = \frac{2.456}{20} = 0.1228$

$\therefore \theta = \tan^{-1}(0.1228) \doteq 7°$ (7.00087)

Note that DC & AO are parallel.

$\therefore \angle AOB = \angle CDB = \theta = 7°$

$\therefore$ Circumference $= \left(\frac{360°}{7°}\right) 778$ km

$= \underline{\underline{40011 \text{ km}}}$

Note: Eratosthenes (267 - 194 BC) (chief librarian at Alexandria & Mathematecian, geographer & astronomer...) used the above method to calculate the earth's circumference to be about 39686 km (*) (24 660 miles)

(* Estimated value!)

Exact values:

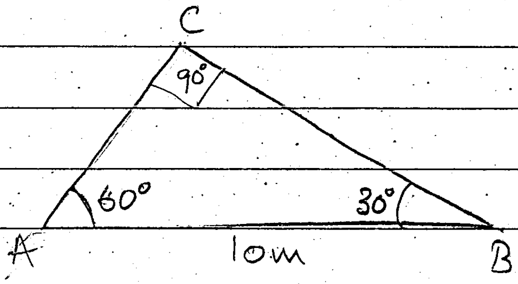
Polar circumference = 40,008 km

Equatorial circumference = 40,075 km

Triangularisation

Triangularisation is a method extensively used in land surveying to establish distances and mapping of large land areas.

Ex - 1



Calculate AC & AB lengths using law of Sines,

$$\frac{AC}{\sin(30°)} = \frac{BC}{\sin(60°)} = \frac{10}{\sin(90°)}$$

$\therefore AC = 10\sin(30°) = 10 \times 0.5$

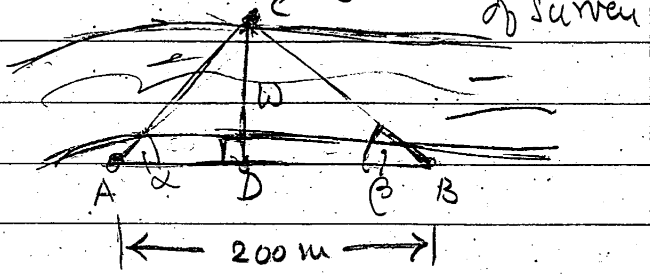
$= \underline{\underline{5 \text{ m}}}.$

$BC = 10\sin(60°) = 10 \times 0.866$

$= \underline{\underline{8.66 \text{ m}}}$

Ex.2

Calculate the width of the river for given values of survey



$\alpha = 32^\circ$ $\quad \beta = 28^\circ$

$\therefore \angle ACB = 180 - (32+28) = 120^\circ \ (\gamma)$

$$\frac{AC}{\sin(\beta)} = \frac{AB}{\sin(\gamma)}$$

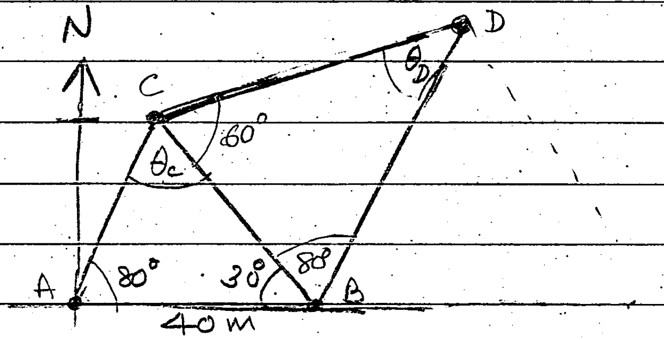
$$\therefore AC = AB \times \frac{\sin(\beta)}{\sin(\gamma)} = \frac{200 \times \sin(28^\circ)}{\sin(120^\circ)}$$

$$= 108.42 \text{ m.}$$

$\therefore$ width of the river

$$= w = AC \sin(\alpha)$$
$$= 108.42 \times \sin(32^\circ)$$
$$= \underline{57.45 \text{ m.}}$$

Ex.3 For The following survey data, calculate the distances AC, BC, CD & BD Given that AB = 40 m



The angle θ_C & θ_D can be calculated as below.

$\Delta ACB: \quad \theta_C = 180 - (80+30) = 70^\circ$

$\Delta CDB: \quad \theta_D = 180 - (60+80) = 40^\circ$

For triangle ACB:

$$\frac{AC}{\sin(30^\circ)} = \frac{BC}{\sin(80^\circ)} = \frac{40}{\sin(70^\circ)}$$

$$\therefore AC = \frac{40 \times \sin(30^\circ)}{\sin(70^\circ)} = \underline{21.28 \text{ m}}$$

$$BC = \frac{40 \times \sin(80^\circ)}{\sin(70^\circ)} = \underline{41.92 \text{ m}}$$

For triangle CDB:

$$\frac{CD}{\sin(80^\circ)} = \frac{BD}{\sin(60^\circ)} = \frac{\overset{BC}{41.92}}{\sin(\underset{\theta_D}{40^\circ})}$$

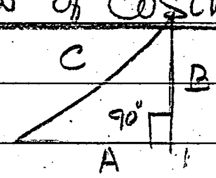
$$\therefore CD = \frac{41.92 \times \sin(80^\circ)}{\sin(40^\circ)} = \underline{64.23 \text{ m}}$$

$$BD = \frac{41.92 \times \sin(60^\circ)}{\sin(40^\circ)} = \underline{56.48 \text{ m}}$$

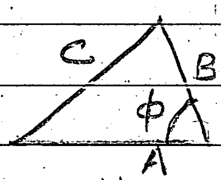
Note: • All distances have been calculated using only angles!!

• Now a days lengths of the triangles are mesured! (Trilateration!)

Law of Cosines



$C^2 = A^2 + B^2$



$C^2 = A^2 + B^2 - 2AB\cos(\phi)$ Valid for any 'ϕ' value

Home work

• Calculate the distance to horizon at (a) h = 10 m (b) h = 400 m

Given That Earth radius = 6378.173 km