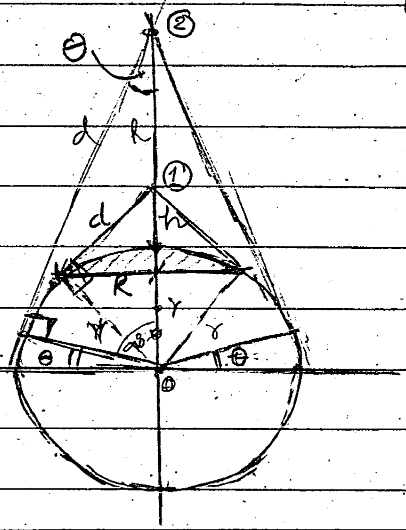
U3A Maths

1-Jul-2025 Term 2 / Week 10

Trigonometry - Examples

∴ We calculated the distance to horizon as below,
We have $r = 6378$ km (radius of earth)



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Considering that the observer is at point (1), we have

$$d^2 = (h+r)^2 - r^2$$
$$= h^2 + 2hr + \cancel{r^2} - \cancel{r^2}$$
$$\therefore d = \sqrt{2hr + h^2}$$

Ex. 1
For height $h = 40$ m ($= 0.04$ km) we have,

$$d = \sqrt{2 \times 0.04 \times 6378 + (0.04)^2}$$
$$= \underline{22.589 \text{ km}}$$

(Note: that "h^2" term has negligible effect on the results)

If required, we can also calculate earth view radius "R". (Homework!)

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- Let us say we move to a higher level, say point (2). Let us choose point (2) by choosing angle θ. For example,
- Choosing $\theta = 20°$ provides a wider view! Note that earth's tilt is 23°.5. Hence, we should be able to see all the continents, except some parts of artic/antartic.

- For convenience, let us derive the equation for 'd' using the angle θ
- Using law of sines

$$\frac{r}{\sin(\theta)} = \frac{(h+r)}{\sin(90°)} = (h+r)$$

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$$\therefore r = (h+r)\sin(\theta)$$
$$= h\sin\theta + r\sin(\theta)$$

$$\therefore h = \frac{r[1-\sin(\theta)]}{\sin(\theta)}$$

Ex. 2 Calculate the height of the observer for $\theta = 20°$

$$h = \frac{6378(1-\sin(20°))}{\sin(20°)}$$
$$= \underline{12{,}270 \text{ km}}$$

Ex. 3 What will be the angle 'θ' when the observer is at height $h = 36{,}000$ km.
(Note: Height of geosynchronous satellite)

we have: $\sin(\theta) = \dfrac{r}{h+r}$

∴ $\sin(\theta) = \dfrac{6378}{36,000 + 6378}$

∴ $\theta = \sin^{-1}\left(\dfrac{6378}{36,000 + 6378}\right) = \underline{\underline{8.656^\circ}}$

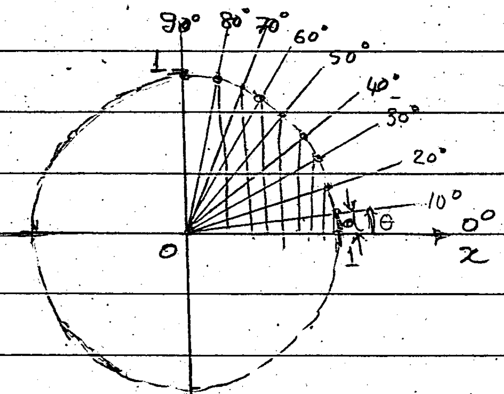
Homework: (1) calculate the "view radius" as a percentage of earth's radius!

(2) what is the "view radius" for space shuttle (h=500km)

• **Sine Function**

we had the question, the sine function corresponds to "half chord" in a circle, hence, why sine function is not a semi circle?

Let us plot the values to check our conjecture!

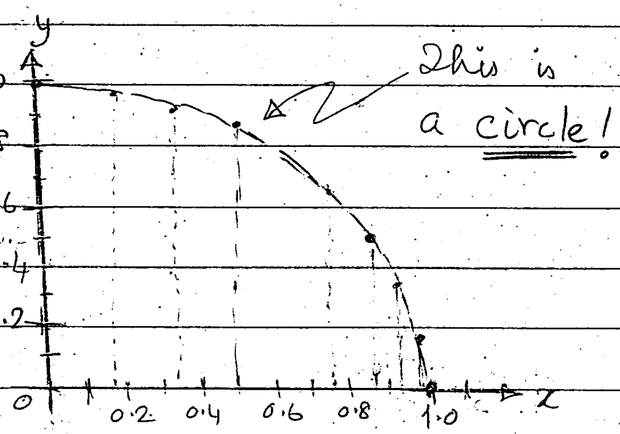


Let us plot the sine values (half chords) as x, y plot for the points shown on the circle, (corresponding to the angles shown).

$\theta \Rightarrow$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$y \Rightarrow$ (sin(θ) values)	0	0.174	0.342	0.5	0.642	0.766	0.866	0.940	0.985	1.0

The corresponding 'x' values are the co-sine values

$x \Rightarrow$	1	0.985	0.94	0.866	0.766	0.642	0.5	0.342	0.174	0.0



• The actual sine function is plotted using the distance

of the points on the circle

— not using 'x' axis values.

• The distance between points on the circle is given by

$d = r\theta$

for our case $r = 1$ ∴ $\underline{\underline{d = \theta}}$

∴ x axis will be 'd' values in radians (along the circle)

θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
"d"	0	.174	.349	.524	.698	.872	1.047	1.221	1.396	1.571 (π/2)
y (sine)	0	.174	.342	.5	.642	.766	.866	.94	.985	1.00