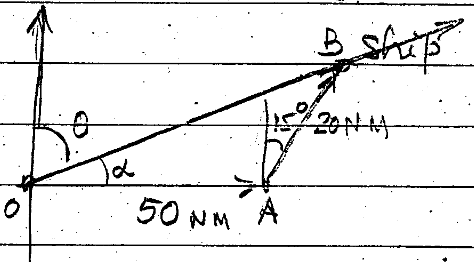
U3A Maths

22-Jul-2025 Term 3 / week 1

Trigonometry – Examples

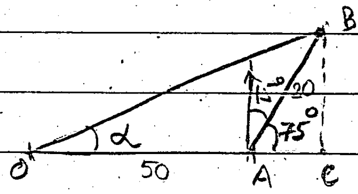
Home work # 1

Calculate the angle 'θ'.
(Bearing of the ship)



NM (Nautical Mile)

Let us find 'α' instead of 'θ'!



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Considering triangle ABC,

$AC = AB \cos(75°) = 20 \cos(75°)$
$= 5.176$ NM

$BC = AB \sin(75°) = 20 \sin(75°)$
$= 19.32$ NM

∴ Considering ΔOBC

$OC = OA + AC = 50 + 5.176$
$= 55.176$ NM

$BC = 19.32$ NM

$\therefore \tan(\alpha) = \dfrac{BC}{OC} = \dfrac{19.32}{55.176} = 0.35$

$\therefore \alpha = \tan^{-1}(0.35) = 19.3°$

Ship's bearing $(\theta) = 90 - \alpha = 70.71°$

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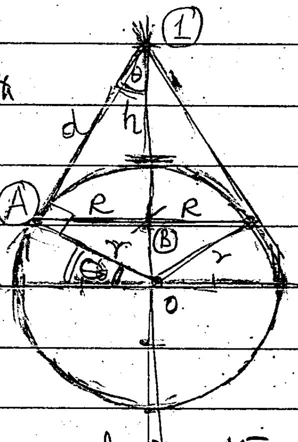
Home work # 2

Given radius of earth
$r = 6378$ km

- We have distance to horizon for height 'h'

$d^2 = (h+r)^2 - r^2$

or $d = \sqrt{2hr + h^2}$



- Alternatively, we can derive the relation between angle 'θ' & 'h', using law of sines

$r / \sin A = (h+r) / \sin(90°)$

$\therefore \sin(\theta) = \dfrac{r}{(h+r)}$

(a) Derive the Equation for The "earth view" radius (R) for an Observer at height 'h'.

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From Δ Ⓐ Ⓑ ①, we have

$R = d \sin\theta$

where

$d = \sqrt{2hr + h^2}$

and $\sin(\theta) = r / (h+r)$

(b) Calculate the view radius (R) for an observer at height h = 500 km. Express the result a % of earth radius.

$d = \sqrt{2 \times 500 \times 6378 + (500)^2}$
$= 2574.5$ km

$\sin\theta = \dfrac{6378}{(6378+500)} = 0.9273$

$\therefore R = d \sin(\theta) = 2574.5 \times 0.9273 = 2387$ km

$\therefore \% R/r \Rightarrow 2387/6378 \times 100 = 37.43\%$

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Ex. 1

Find the value of angle 'x' for which $\sin(x) = \sin(2x)$

(Assume that $x = 0$ to 2π rad.)

We have:

$$\sin(A+B) = \sin(A)\cos(B) + \cos A.\sin(B)$$

$$\therefore \sin(2A) = 2.\sin(A)\cos(A)$$

$$\therefore \sin(2x) = 2.\sin(x)\cos(x)$$

If we have

$$\sin(x) = \sin(2x) \Rightarrow \text{Given}$$

$$\therefore \cancel{\sin(x)} = 2\cancel{\sin(x)}\cos(x)$$

$$\therefore \cos(x) = \tfrac{1}{2} \quad \therefore x = \pi/3 \text{ or } 60^\circ$$

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Ex. 2

Show that

(a) $\sin(90^\circ - A) = \cos(A)$

& (b) $\cos(90^\circ - A) = \sin(A)$

We have

$$\sin(90^\circ - A) = \sin(90^\circ)\cos(A) - \cancel{\underbrace{\cos(90^\circ)}_{(=0)}.\sin(A)}$$

$$= \sin(90^\circ)\cos(A) = \cos(A)$$

$$\cos(90^\circ - A) = \cancel{\overbrace{\cos(90^\circ)}^{(=0)}}.\cos(A) + \sin(90^\circ)\sin(A)$$

$$= \sin(90^\circ).\sin(A)$$

$$= \sin(A)$$

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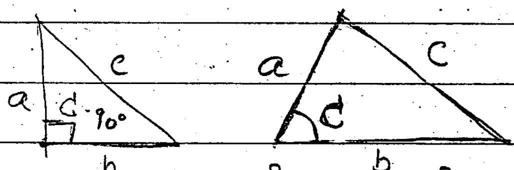
Law of cosines

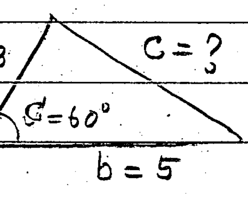


$c^2 = a^2 + b^2$ $\qquad$ $c^2 = a^2 + b^2 - 2ab\cos(C)$

(Note: Angle 'C' is opposite to side 'c')

Ex. 3

Find the value of side 'c' for the triangle shown below. Check the results using "law of sines", by calculating the angles.



$$c^2 = a^2 + b^2 - 2ab\cos(C) \quad a = 3$$

$$= 3^2 + 5^2 - 2\times3\times5\times\cos(60^\circ)$$

$$= 9 + 25 - 30\times\tfrac{1}{2}$$

$$= 19 \quad \therefore c = \sqrt{19} = 4.359$$

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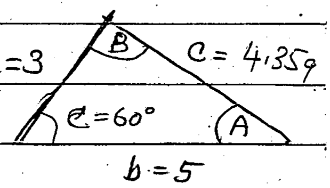
Check using law of sines

We have

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)}$$



$$\frac{4.359}{\sin(60^\circ)} = \frac{3}{\sin(A)}$$

$$\therefore \sin(A) = \frac{3.\sin(60^\circ)}{4.359} = 0.596$$

$$\therefore A = \sin^{-1}(0.596) = 36.58^\circ$$

Also $\dfrac{4.359}{\sin(60^\circ)} = \dfrac{5}{\sin(B)}$

$$\therefore \sin(B) = \frac{5\times\sin(60^\circ)}{4.359} = 0.9934$$

$$B = \sin^{-1}(0.9934) = 83.41^\circ$$

Check $A + B + C = 36.58^\circ + 83.41^\circ + 60^\circ$

$$= 179.99 \approx 180^\circ \checkmark$$