U3A Maths

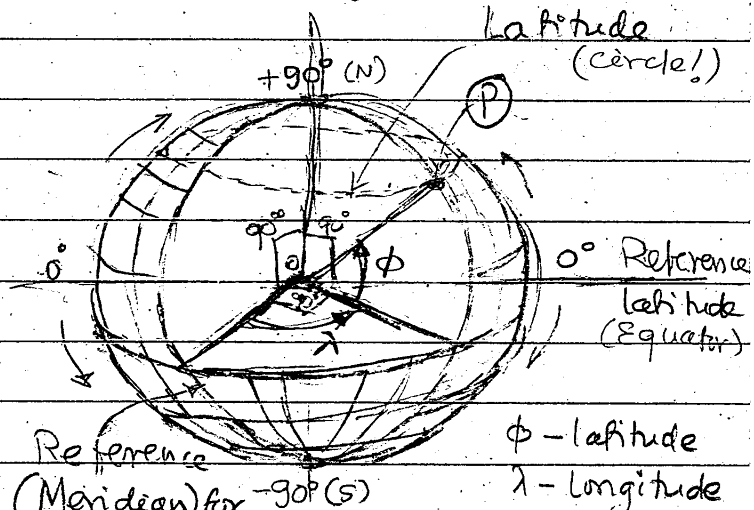
29-Jul-2025 Term 3 / Week 2

Latitude

- The main motivation for trigonometry was the astronomical observations, however, it also helped advancements in navigation.

- After recognising that the earth is spherical, it was found useful to use "angles" instead of "linear lengths" to locate the points on earth and consequently for navigation.

- Latitudes & Longitudes are imaginary lines on earth's surface, which enable location and navigation.
- The best way of illustrating latitudes & longitudes is shown below



- The location of point 'p' on the sphere (earth) can be specified by angles ϕ & λ, ϕ - latitude ; λ - longitude.
- Where, ϕ is the "latitude" angle using equitorial surface as the reference. Note, that angle 'ϕ' specifes a "circle" around the earth.

$\phi = 0° \Rightarrow$ Equator

$\phi = +90° \Rightarrow$ North pole (90° N)

$\phi = -90° \Rightarrow$ South pole (90° s)

Earth's tilt $\Rightarrow \phi = +23.5 \Rightarrow$ Tropic of Cancer (23.5 N)

$\phi = -23.5 \Rightarrow$ Tropic of Capricorn (23.5 s)

$\phi = (90 - 23.5) = +66.5 \Rightarrow$ Artic circle (66.5 N)

$\phi = (-90-(23.5)) = -66.5 \Rightarrow$ Antartic circle (66.5 S)

(Note: We will discuss Longitude details in a separate lecture).

- How to determine the latitude of a given location?
- This can be done using a sextant and a reference star.
  - Pole star in the Northern hemisphere (Easy!)
  - Southern star and southern pointer stars are used to locate south pole (Difficult)
- Latitude can also be determined by Sun's position at Noon. But, the earth's declination angle needs to be considered.
- Earth declination angles for Northern hemisphere seasons are:

Jun 22 $\Rightarrow$ 23.5 $\rightarrow$ summer Solstice

Mar 21 & Sep 23 $\Rightarrow$ 0° $\rightarrow$ Equinox

Dec 22 $\Rightarrow$ -23.5 $\rightarrow$ Winter solstice

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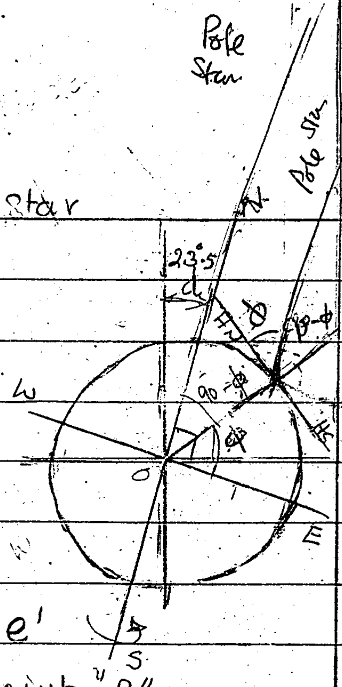

Latitude using Pole Star

- This can only be used in Northern Hemisphere.
- Angle of elevation of pole star w.r.t. (North) horizon — let it be called 'e'
- Latitude of point "P"

$$\phi = e$$

- Note that earth's declension or "tilt" (23.5°) does not affect the measurement, since 'Pole star' is directly above the North Pole. ($\simeq 89.35$)
- There is no such bright star above the South Pole.

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- Hence, The above method is not popular in Southern hemisphere.
- A more versatile method is to measure the elevation of the Sun at Noon.
- This method is applicable for both Northern and Southern hemisphere.
- However, we need to consider varying declension w.r.t. Sun depending on the season.
- The declension value can be established, based on calender days using Equinox day (say Mar 21) as our reference.

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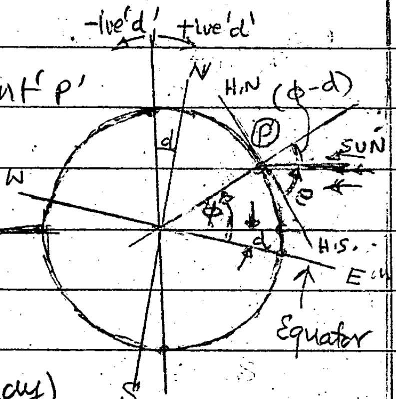
Latitude using Sun's Position



- Let us consider point 'P' in the Northern hemisphere.
- The measurement is made at Noon. (Mid-day)
- Using southern horizon as reference, the angle of elevation of Sun is 'e' w.r.t. (South) Horizon
- Referring to the Figure

$$(\phi - d) + e = 90^\circ$$

$\therefore$ Latitude $\phi = (90^\circ + d - e)$

- Notes: (1) Declension angle (d) is positive in clockwise direction

(2) Sun elevation angle (e) is w.r.t. Horizon-South (H-S)

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- The declension angle 'd' in degrees can be calculated using the formula

$$d = 23.5° \sin\left(\frac{360}{365} \times n\right)$$

where 'n' is the number of days after Mar 21 (Northern Spring Equinox declension = 0°)

- For March 21, $n = 0$, hence declension 'd' = 0° ⇒ Equinox day (Equator directly facing sun)

Alternatively,

$$d = 23.5 \sin\left(\frac{360 \times (285 + N)}{365}\right) \text{deg}$$

where, 'N' is "Day of the Year" (N = 1 to 365)

Note: Mar 21 is 80th day of the year $\left(\begin{matrix}285+80 \\ = 365\end{matrix}\right)$