U3A Maths

12-Aug-2025 Term3/Week4

Latitude from shadow

Review

Equations for Sun's elevation angle (e) [often called 'Altitude'] at Mid day

Eqn(1): $e = 90° + d - \phi$ ⇒ Northern Hemisphere

Eqn(2) $e = 90° - d - \phi$ ⇒ Southern Hemisphere

where

ϕ ⇒ Latitude.

d ⇒ Earth's declension angle relative to Sun

$$= 23.5 \times \sin\left(360 \times \frac{(285+N)}{365}\right)$$

where N – Day of the Year

Jan 1 ⇒ N=1

Dec 31 ⇒ N=365

Ex.1 Calculate the Sun's elevation angle at midday for Latitude (ϕ) = 70° N, for Dec 22nd (Northern Hemisphere Winter Solstice)

we have (Northern Hemisph)

$\phi = 70°$ (Use Eqn.1)

$N = 356$ (For Dec. 22)

$$d = 23.5 \times \sin\left(360 \times \frac{(285+356)}{365}\right)$$

$$= -23.48$$

Using Eqn 1

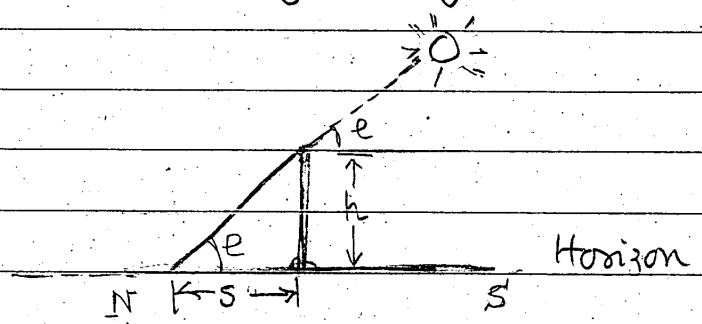
$$e = 90° + d - \phi$$

$$= 90° + (-23.5) - 70°$$

$$= -3.5°$$

The Sun is below the horizon !!

- Sun's elevation angle can ~~also~~ be established by the length of the shadow.
- Note: Eqn (1) & (2) can only be used for the shadow at Mid day (of any given day!)



Note: The Sun is above the Horizon South for Northern Hemisphere and Vice Versa for Southern Hemisphere.

we know the height of the pole (h) and we can measure length of shadow (s)

we have $\tan(e) = \frac{h}{s}$

Sun's elevation angle

$$e = \tan^{-1}\left(\frac{h}{s}\right)$$

Hence,

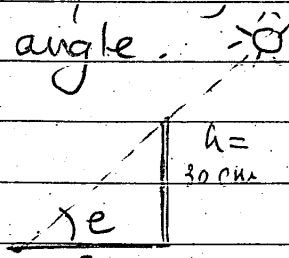
Latitude of the place,

$\phi = 90° + d - e$ ⇒ (Eqn 1) for Northern Hemisphere

For Southern Hemisphere,

$\phi = 90° - d - e$ ⇒ (Eqn 2)

Ex.2 A 30 cm pole casts a shadow of length 40 cm. What is the corresponding Sun's elevation angle.



$$\tan(e) = \frac{h}{S} = \frac{30}{40}$$

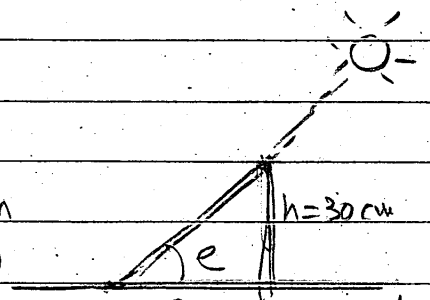
$$= 0.75$$

$$e = \tan^{-1}(0.75)$$

$$= 36.87^\circ$$

Ex.3 The shadow measured in Derby st., Penrith on 6-Aug-2025 at approximately midday was 37 cm for a pole of height 30 cm

Calculate the latitude of Penrith.



We have

Sun's elevation

$$e = \tan^{-1}(h/S)$$

$$= \tan^{-1}(30/37)$$

$$= 39.04^\circ$$

Declension angle (d) for 6-Aug-2025

$N = 218$

$$d = 23.5 \sin\left(360 \times \frac{(285+218)}{365}\right)$$

$$= 16.29^\circ$$

∴ Penrith latitude using Eqn.(2)

$$\phi = 90^\circ - d - e = 90^\circ - 16.29 - 39.04$$

$$= 34.67^\circ$$

[As per GPS Penrith Latitude = 33.76°]

Home Work

(1) For the latitude of 23.5°S, find the day of the year for which there is no shadow at mid-day, i.e, the Sun's elevation angle is 90°.

(2) For the place located at latitude 30°N, find the length of the shadow at mid day on winter Solstice day in Northern hemisphere (Dec 22)

(3) Let us improve the measurement made in Ex.3. Measure the shadow at midday on 17th Aug 25 (sunday) and calculate the Latitude of your place. [we can compare notes in the next class on 19th Aug 25]