U3A Maths

19-Aug-2025 Term 3 / Week 5

Sun's Elevation Angle (Altitude)

Home work #1

Given: Latitude $\phi = 23^\circ.5\,S$

Find the day of the year (N) for which there is no shadow at mid-day ie, Sun's elevation $e = 90^\circ$

Soln

$23^\circ.5\,S$ is Southern hemisphere (Tropic of Capricorn!)

hence use Eqn(2)

$$\phi = 90^\circ - d - e$$

-2-

we have

$$\text{Eqn(2)} \Rightarrow 23^\circ.5 = \cancel{90^\circ} - d - \cancel{90^\circ}$$

$$\therefore d = -23^\circ.5$$

ie, $$-23.5 = 23.5\,\text{Sin}\left(\frac{360 \times (285+N)}{365}\right)$$

$$\therefore \frac{360(285+N)}{365} = \text{Sin}^{-1}\left(-\frac{23.5}{23^\circ.5}\right)$$

$$= \text{Sin}^{-1}(-1)$$

$$= -90^\circ \text{ or } 270^\circ$$

$$\therefore (285+N) = -90 \times \frac{365}{360} = -91.25$$

$$\therefore N = -285 - 91.25 = -376.25 \quad (>365)$$

$$N = -376.25 + 365 = -11.25$$

$$\therefore N = \text{Dec}(31 - 11.25) \approx \text{Dec } 20^{th}$$

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Note: Summer Solstice in Australia occurs between Dec 21 to 23.

{About 0.5% error?}

Home Work #2

For the place located at 30°N, find the length of the shadow of 30cm pole at midday on Dec 22 (winter Solstice in Northern hemisphere)

For Dec 22 $\Rightarrow N = 356$

$$\therefore d = 23.5 \times \text{Sin}\left(\frac{360 \times (285+358)}{365}\right)$$

$$= -23.48 \text{ or } -23.5$$

(Winter Solstice!)

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$\phi = 30^\circ N \Rightarrow$ Northern Hemisphere

$\therefore$ Use Eqn(1)

$$\text{Eqn(1)} \Rightarrow \phi = 90^\circ + d - e$$

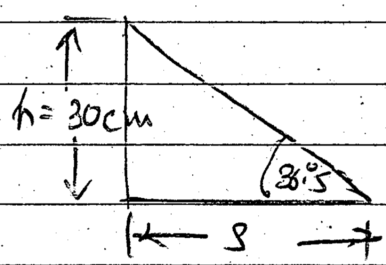
We have $30^\circ = 90^\circ - 23.5 - e$

$$\therefore e = 90^\circ - 23^\circ.5 - 30^\circ$$

$$= \underline{36^\circ.5}$$

$\therefore$ the shadow length is

$$\tan(36^\circ.5) = \frac{h}{s} = \frac{30}{s}$$



$$\therefore s = \frac{30}{\tan(36^\circ.5)} = \underline{40.54 \text{ cm}}$$

Home work # 3

Discuss the results in class!

Ex.1 For latitude of 50° N find the sun's elevation angle for:

(a) Equinox Day

(b) Summer Solstice

(c) Winter Solstice.

We have $\phi = 50°$ (N)

Northern Hemisphere ∴ Use Eqn(1)

$$\phi = 90° + d - e$$

(a) $d = 0°$ – Equinox

$$\therefore \; e = 90° + d - \phi = 90 + 0 - 50°$$
$$= \underline{40°}$$

(b) $d = +23.5°$ – Summer Solstice

$$\therefore \; e = 90° + d - \phi$$
$$= 90° + 23.5° - 50°$$
$$= \underline{63.5°}$$

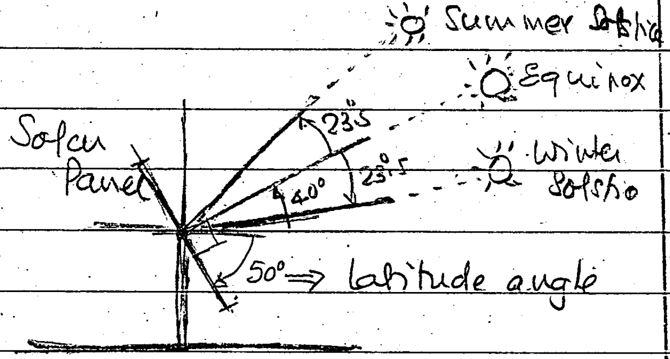
(c) $d = -23.5°$ – Winter Solstice

$$e = 90° + (-23.5°) - 50°$$
$$= \underline{16.5°}$$

Solar Panel Angle

- The ideal angle for maximum solar power, the panels are positioned perpendicular to the sun's rays.

- The ideal angle for solar panels is usually equal to the latitude of the location.



- Hence, the ideal angle for

  Sydney ⇒ 34°

  Melbourne ⇒ 38°

  Brisbane ⇒ 27°

  Perth ⇒ 32°

- However the solar panel angle are also chosen, so as to

maximise solar power for a given season for example, maximising solar power in winter.

- As per an on-line calculator, the optimal angles for Penrith are as below:

  Latitude of Penrith = 33.755°

  Optimal for the year ⇒ 29.4°

  Optimal for

  Spring ⇒ 29.4°

  Summer ⇒ 44.4°

  Autumn ⇒ 29.4°

  Winter ⇒ 14.4°

Jan – 19.4°	Apr – 34.4°	Jul – 39.4°	Oct – 24.4°
Feb – 24.4°	May – 39.4°	Aug – 34.4°	Nov – 19.4°
Mar – 29.4°	Jun – 44.4°	Sep – 29.4°	Dec – 14.4°