U3A Maths

26-Aug-2025 Term 3 / Week 6

Latitude Calculations - REVISED

- We have been using the equation for declination angle as below:

$$d = 23.5 \times \sin\left(\frac{360 \times (285+N)}{365}\right) \quad (*)$$

[I am not sure where I picked up "285", but all the literature use the figure "284"!]

- In future, let us use the following

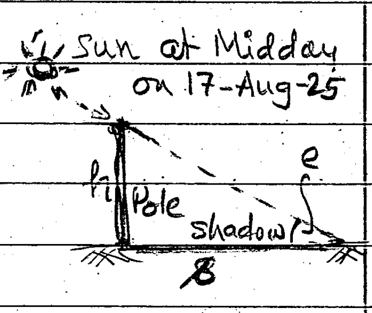
CORRECT equation with higher precision.

- Earth's Declination Angle (d) at Mid-day (Solar Noon)

N ⇒ day of the Year (Jan 1 = 1)

$$\boxed{d = 23^\circ.45 \times \sin\left(\frac{360 \times (284+N)}{365.25}\right)}$$

- Let us also do the calculations for a minimum of 4 decimal digits.
- For leap year, use 366.25 instead of 365.25
- Let us now revise the latitude calculations (Homework #3), using Phil Timbrell's experimental results.



17-Aug-25 ⇒ N = 229

We have

$h = 1810$ mm

$s = 1940$ mm

∴ Sun's elevation angle (e)

$$e = \tan^{-1}\left(\frac{h}{s}\right) = 43^\circ.0145$$

Declination (d)

$$d = 23^\circ.45 \sin\left(\frac{360 \times (284+N)}{365.25}\right)$$

$$= 23.45 \sin\left(\frac{360 \times (284+229)}{365.25}\right)$$

$$= 13^\circ.2396$$

For the Southern Hemisphere,

$$\text{Latitude } \phi = 90^\circ - d - e$$

$$= 90^\circ - 13^\circ.2396 - 43^\circ.0145$$

$$= 33^\circ.7459 \text{ S}$$

- This compares well with the following latitude values.

$\phi_{Penrith} = -33.757215$

$\phi_{Glenbrook} = -33.76384$!!

- We have used the following equations for the latitude,

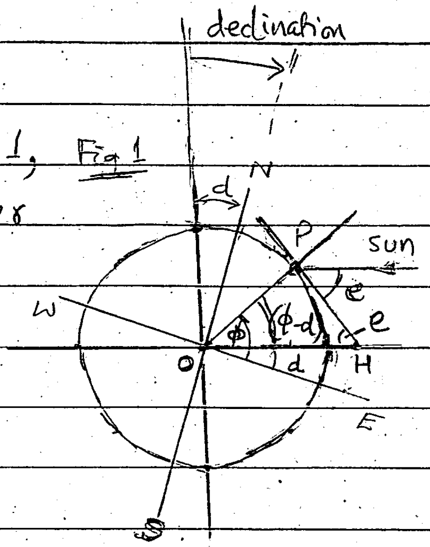
Eqn(1) ⇒ $\phi = 90^\circ + d - e$ ⇒ Northern Hemisphere

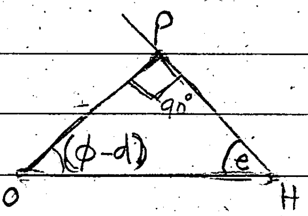
Eqn(2) ⇒ $\phi = 90 - d - e$ ⇒ Southern Hemisphere

- We can derive alternative equations for 'ϕ', using trigonometric functions!

- Refering to Fig 1, let us consider the triangle "OPH"

Fig 1





Using the "Law of Sines" for ΔOPH

$$\frac{OP}{\sin(e)} = \frac{OH}{\sin(90°)}$$

Also, $\cos(\phi-d) = \dfrac{OP}{OH}$

$\therefore \quad OP = OH \cdot \cos(\phi - d)$

Hence, $\dfrac{\cancel{OH} \cdot \cos(\phi-d)}{\sin(e)} = \dfrac{\cancel{OH}}{\sin(\cancel{90°})_{(1)}}$

$$\therefore \sin(e) = \cos(\phi - d)$$

Note that $\cos(-\theta) = \cos(\theta)$

Hence,

$\sin(e) = \cos(d - \phi)$ is also valid!

Eqn(1) $\Rightarrow \sin(e) = \cos(\phi - d)$ — Northern Hemisphere

Eqn(2) $\Rightarrow \sin(e) = \cos(d-\phi)$ — Southern Hemisphere

Ex. 1 Use trigonometric Eqns to calculate the Latitude for Phil Timbrell's results.

We have: $h = 1810$ & $S = 1940$

$$e = \tan^{-1}(h/s) = 43°.0145$$

$$d = 13°.2396 \text{ for } 17\text{-Aug-}25$$

Using Eqn(2) for Southern Hemisphere.

$$\sin(e) = \cos(d - \phi)$$

$$\sin(43°.0145) = \cos(13°.2396 - \phi)$$

i.e., $0.6822 = \cos(13°.2396 - \phi)$

$$\therefore (13°.2396 - \phi) = \cos^{-1}(0.6822)$$

$$= 46°.9842$$

$\therefore \phi = 13.2396 - 46.9842 = -33°.7446$

Note: Negative value / Southern Hemisphere!!

Ex. 2 In Philadelphia (US.), the sun's elevation angle was measured to be 30° on 21st Jan at Solar Noon. Calculate the latitude.

we have, $e = 30°$

$$d = 23°.45 \sin\left(\frac{360 \times (284 + 21)}{365.25}\right)$$

$$= -20°.1811$$

Using Eqn (1) (for the Northern Hemisphere)

$$\sin(e) = \cos(\phi - d)$$

$$\sin(30°) = \cos(\phi - (-20°.1811))$$

$$0.5 = \cos(\phi + 20°.1811)$$

$$(\phi + 20°.1811) = \cos^{-1}(0.5) = 60°$$

$$\therefore \phi = 60° - 20°.1811$$

$$= +39°.8189$$

Note: Positive Angle! Northern Hemisphere!