U3A Maths

5-Aug-2025 Term 3/Week 3

Latitude - Examples

Review - Latitude using Sun's position.

- Measurement is done at Noon.
- Declension angle (d) of earth relative to the Sun varies from $d_1 = +23.5$ to -23.5 depending on the day of the year.

[Mar 21 & Sep 21 $\Rightarrow d = 0°$]

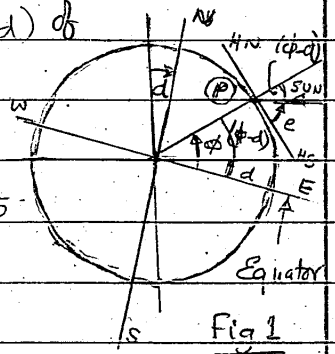


Fig 1

- Mar 21 - Jun 22 - Sep 21 $\Rightarrow d = 0°$ to $+23.5°$ to $0°$ Northern Summer
- Sep 21 - Dec 22 - Mar 21 $\Rightarrow d = 0$ to -23.5 to $0°$ Southern Summer

- The general equation for declension angle (d):

$$d = 23.5 \times \sin\left(\frac{360° \times n}{365}\right)$$

where n - No. of days after Mar 21

Alternatively,

$$d = 23.5 \times \sin\left(\frac{360° \times (285+N)}{365}\right)$$

where N - is the "Day of the Year"

Northern Hemisphere (Refer Fig 1)

$(\phi - d) + e = 90°$

$\therefore$ Latitude $= \phi = (90° + d - e)$ - Eqn(1)

where e is the angle of elevation of Sun w.r.t. "Horizon South" (H-S)

Southern Hemisphere (Refer Fig. 2)

- In the southern Hemisphere, since the Sun is on the Northern Horizon, the Sun's elevation angle (e) is measured facing North. Hence, elevation angle (e) is measured w.r.t. Horizon North (H-N)

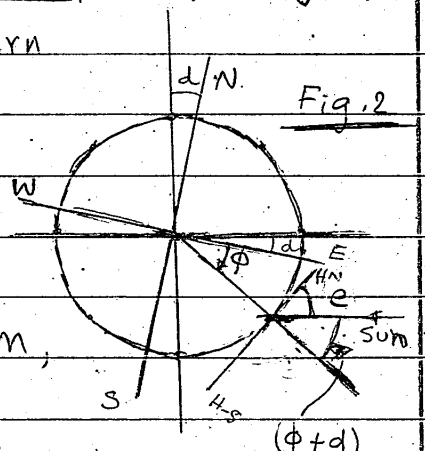


Fig. 2

- From Fig 2, we have

$(\phi + d) + e = 90°$ - Eqn(2)

$\therefore \phi = (90° - d - e)$

- Note: ϕ values for southern hemisphere are expressed

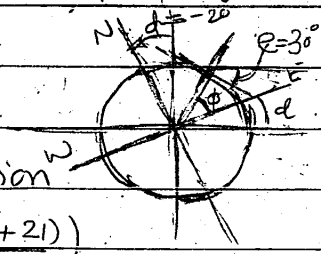
as "negative" angle or with suffix "S".

That is, if the calculated value of 'ϕ' from Eqn.(2) is $40°$. It is expressed as $40°S$ or $-40°$

Ex. 1

In Philadelphia (U.S.) the angle of elevation (often called Altitude!) of the Sun (w.r.t. Southern horizon) was measured to be $30°$. The measurement was made on 21st Jan at Noon. Calculate the Latitude of the place.

Given date is 21st Jan hence,
"Day of the year"
N = 21



∴ Earth's declination

$$d = 23.5 \sin\left(\frac{360 \times (365+21)}{365}\right)$$

$$= -19.97 \cong -20°$$

∴ Using Eqn.(1), with $e = +30°$

Latitude $\phi = 90° + d - e$

$= 90° + (-20°) - 30°$

$= 90° - 50° = +40°$

The latitude of Philadelphia is 40° N or +40°.

Ex. 2 (Home work problem).

At Rockhampton (latitude is 23.5 S or −23.5), calculate the elevation of the Noon Sun on

(a) Mar 21 (b) Dec. 22 (c) Jun 5

(a) Mar 21

Day of the year

N = 31 + 28 + 21 = 80

Earth's declination

$$d = 23.5 \times \sin\left(\frac{360 \times (285+80)}{365}\right)$$

$= 0°$

We have Latitude = −23.5 or 23.5 S

Using Eqn (2): $\phi = 23.5$

We have: $\phi = 90° - d - e$

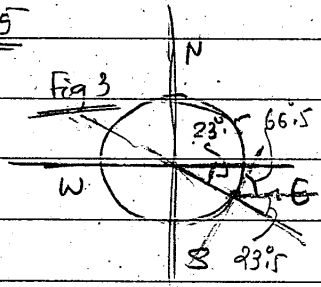
$23.5 = 90 - 0 - e$

∴ $e = 90 - 23.5$

$= 66.5$

(See fig (3))

Fig 3



(b) Dec. 22

Day of the year N = 356

∴ Earth's declination

$$d = 23.5 \times \sin\left(\frac{360 \times (285+356)}{365}\right)$$

$= -23.5$

∴ Using Eqn (2) & $\phi = 23.5$

$\phi = 90° - d - e$

$23.5 = 90° - (-23.5) - e$

∴ $e = 90°$

Sun is directly above at Noon. (Summer Solstice.)

(c) Jun 5

Day of the year (N) = 156

∴ Earth's declination $d = +22.7$

Using Eqn (2) and $\phi = 23.5$ we have,

$\phi = 90° - d - e$

$23.5 = 90 - 22.7 - e$

∴ $e = 90 - 22.7 - 23.5$

$= 43.8$