U3A Maths

23-Sep-2025 Term 3/Week10

Solar Time

- We have the equation for for sun's elevation angle at Solar Noon, as below:

$\sin(e) = \cos(\phi - d) \Rightarrow$ Northern Hemi

$\sin(e) = \cos(d - \phi) \Rightarrow$ Southern Hemi

where,

e – sun's elevation angle at Solar Noon

ϕ – Latitude of the location

d – earth's declination (depends on day of the year)

Note: For given ϕ & d; both equations give the same results for 'e'.

- Solar Noon occurs when the sun is due South (Northern Hemi), due North (Southern Hemi) or directly above.
- Solar Noon also results in shortest shadow!
- We can also write:

$$\sin(e) = \cos(\phi - d) = \sin(\phi).\sin(d) + \cos(\phi).\cos(d)$$

$\{$Note: $\cos(A-B) = \cos(A).\cos(B) + \sin(A)\sin(B)\}$

- We can generalise the above equation to obtain solar elevation angle (e) at any given time – not just ~~for~~ solar Noon!

- Such an equation is

$$\sin(e) = \sin(\phi).\sin(d) + \cos(\phi).\cos(d).\cos(h)$$

→ Eqn (1)

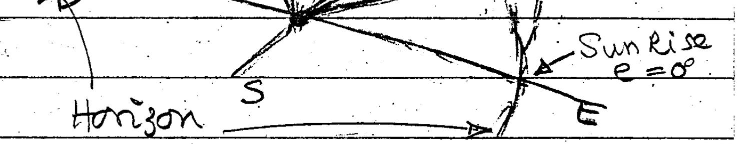
Where "h" is called the "Solar Hour Angle (SHA)"

- For $h = 0° \Rightarrow \cos(h) = \cos(0°) = 1$

∴ For $h = 0°$, the above equation corressponds to Solar Noon!

Note: The above generalised equation is deceptively simple. The derivation of the equation requires spherical trigonometry and vector calculus! we are not going there!!

- Concept of Solar Hour Angle (h)



- In the Figure, the sun is due North, hence it corresspnds to Southern Hemisphere.
- Solar Noon (12 Noon) $\Rightarrow h = 0°$

h is +ive for (Solar) "After Noon"

h is −ive for (Solar) "Before Noon"

- For a full day, sun moves 360°. Hence 'h' values vary from 0 to +180° and 0 to −180°

- It is interesting to note that Sun's elevation angle is zero ($e = 0°$) at Sun rise and Sun set.
- The angle "e" is negative during the night, that is, the sun is below the horizon!
- Note: It is important to understand the concept of 'h' and 'e', by studying the Figure closely.
- The solar hour angle (h) can be converted to Solar Time (ST)

$$ST = 12 + h°/15° \Rightarrow \text{Eqn } ②$$

- The rotation of earth's meridian (Longitude) corresponds to 15° per hour (earth rotation is $360°/24\,hrs = 15°/hour$)
- For sun rise & sun set, we have sun's elevation angle (e) $= 0°$

$$\sin(e) = \sin(\phi)\sin(d) + \cos(\phi).\cos(d).\cos(h_s) = 0$$

Where, h_s – Solar Hour Angle for sun rise / sunset

$$\cos(h_s) = -\frac{\sin(\phi)\sin(d)}{\cos(\phi)\cos(d)}$$

Hence, $\cos(h_s) = -\tan(\phi).\tan(d)$

$$\therefore h_s = \cos^{-1}\left[-\tan(\phi).\tan(d)\right] \Rightarrow \text{Eqn } ③$$

$\therefore$

$$\text{Solar Time (sunset)} = 12 + h_s°/15°$$

$$\text{Solar Time (Sun Rise)} = 12 - h_s°/15°$$

—x—

Ex.1 What is the Solar Hour Angle for the following Solar times.
(a) 12 Noon (b) 1 PM (c) 2 pm
(d) 11 am (e) 9 am (f) 7 pm

Using Eqn ②;

$ST = 12 + h/15 \quad \therefore h = (ST - 12) \times 15$ deg

(a) 12 Noon $h = (12-12)\times 15 = 0°$
(b) 1pm $h = (13-12)\times 15 = 15°$

(c) 2 pm $\Rightarrow h = (14-12)\times 15 = 30°$
(d) 11 am $\Rightarrow h = (11-12)\times 15 = -15°$
(e) 9 am $\Rightarrow h = (9-12)\times 15 = -45°$
(f) 7pm $\Rightarrow h = (19-12)\times 15 = 105°$

Ex.2
What is the sun's elevation angle (e) on 23rd Sep 2025 at Ingleburn (NSW) [$\phi = -34°$] at 2 PM Solar Time.

Latitude $\phi = -34°$

Solar Time 2PM (14 hrs) $\Rightarrow h = (14-12)\times 15° = +30°$

23rd Sep $\Rightarrow N = 266$

$\therefore$ Earth declination

$$d = 23.45 \times \sin\left(\frac{360 \times (284+266)}{365.25}\right) = -0.857°$$

Using Eqn (1)

$$\sin(e) = \sin(\phi).\sin(d) + \cos(\phi).\cos(d)\cos(h)$$

$$= \sin(-34^\circ).\sin(-0.857^\circ) + \cos(-34^\circ)\cos(-0.857^\circ).\cos(30^\circ)$$

$$= \underline{0.7263}$$

∴ Sun's elevation Angle

$$e = \sin^{-1}(0.7263)$$
$$= \underline{46.58^\circ}$$

Ex. (3) For Ex. 2, what is the Sunset (Solar) time.

At Sunset $e = 0$

The Solar Hour Angle is:

$$\therefore \cos(h_s) = -\tan(\phi)\tan(d)$$

$$= -\tan(-34^\circ).\tan(-0.857^\circ)$$
$$= -0.01^\circ$$

$$\therefore h_s = \cos^{-1}(-0.01^\circ) = \underline{90.57^\circ}$$

∴ Sunset (Solar) Time

$$= 12 + h_s/15 = 12 + \frac{90.57}{15}$$
$$= 18.038 \text{ hrs} \text{ or } \underbrace{6}_{hrs} : \underbrace{2.28}_{Min} \text{ pm}$$

Notes

- The actual sun set time (clock time) is 5:53 pm
- We need to develop a correction factor to convert Solar Time to clock time at a given location.
- We will do this in the next few weeks.

Home work

Calculate the following at Kansas City ($\phi = +39.3^\circ$) on February 15th.

(a) Sun rise Solar Hour Angle and the Solar time

(b) Sunset Solar Hour Angle and the Solar time.