U3A Maths

2-Sep-2025 Term3 / Week7

Longitude

Review: In the last class we derived alternative Equations for Sun's elevation, latitude & earth's decl.

Eqn(1) ⇒ $\sin(e) = \cos(\phi - d)$ — Northern Hemi.

Eqn(2) ⇒ $\sin(e) = \cos(d - \phi)$ — Southern Hemi

Also, we revised the Equation for Earth's declination angle:

$$d = 23.45 \sin\left(\frac{360 \times (284 + N)}{365.25}\right)$$

where $N = 1$ — for Jan 1
$= 365$ — Dec 31

- Further investigation (!!) by Jesha revealed that Eqn (1) can be used for both Northern & Southern hemispheres with appropriate sign for Latitude!
Let us check it out

Ex.1 Calculate the Sun's elevation angle at Mid-day (Solar Noon) on 22nd Dec. at latitudes

(a) $\phi = +23.45$ — Tropic of Cancer
(b) $\phi = -23.45$ — Tropic of Capricorn

we have
$N = 356$ (Dec 22)

$$\therefore d = 23.45 \sin\left(\frac{360 \times (284 + 356)}{365.25}\right)$$

$= -23.4477$ (say -23.45)

(a) $\phi = +23.45$

$\therefore \sin(e) = \cos(\phi - d)$
$= \cos(23.45 - (-23.45))$
$= \cos(46.9)$
$= 0.6833$
$\therefore e = \sin^{-1}(0.6833)$
$= 43.1$

(b) $\phi = -23.45$

$\sin(e) = \cos(\phi - d)$
$= \cos(-23.45 - (-23.45))$
$= \cos(0) = 1$
$\therefore e = \sin^{-1}(1) = 90°$

Ex.2 Calculate the Sun's elevation angle at mid-day (Solar Noon) on 22nd Jun at latitudes:

(a) $\phi = +75°$ (b) $\phi = -75°$

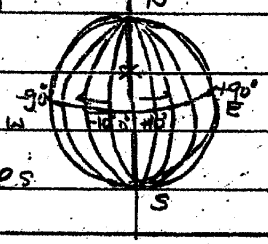
we have $N = 173$ for Jun 22nd
$\therefore d = +23.4494$ (say 23.45)

(a) $\phi = +75°$ (Northern Hemi)
$\sin(e) = \cos(75° - 23.45) = \cos(51.55)$
$= 0.6218$
$\therefore e = \sin^{-1}(0.6218) = 38.45$

(b) $\phi = -75°$ (Southern Hemi)
$\sin(e) = \cos(-75 - 23.45)$
$= -0.147$
$e = \sin^{-1}(-0.147) = -8.453$
Sun is below horizon!

Longitude

- Longitudes are imaginary lines which run vertically from North pole to South pole.
- They are also called "Meridians"
- An (arbitrary) reference longitude is called the Prime Meridian.
- The longitude passing thru Greenwich (U.K.) was chosen as the Prime meridian in 1884 as the International Standard.
- The prime meridian is assigned 0°
- Since, earth is a sphere, the longitudes



are assigned 0° to +180° East of Prime Meridian and 0 to -180° West of the prime Meridian.

- A given value of latitude & longitude specifies exact location of a place on the globe or a map
- Historically, establishing the longitude of a place was more challenging. Many Sailors in 1600s & 1700s lost their life due to the difficulty of establishing longitude. In one incident in 1707, over 1,000 British Soldiers died as a result of navigational error.

- In 1714 a prize of £20,000 (by Queen Anne) (Approx US$ 4 Million) was announced to produce a practical method to determine the longitude.
- The earth takes 24 hours to make one full rotation. This corresponds to 360°.
- Hence, the earth moves at 360°/24 hr = 15°/hr of latitude.
- Therefore, relative time w.r.t. to prime Meridian (GMT or UTC) at a given location indicates the latitude of the place.
- John Harrison (1693-1776) developed the "Chronometer" after several trials, which became a Standard device to establish longitude

- In practice, every 15° latitude is assigned a time zone w.r.t. GMT.

Ex.1 Clock 'A' carried by a Sailor is set to GMT. At the "local" Noon the clock reads 11 am. What is the longitude of the location?

Local
Since the Noon (12'o clock) is one hour ahead of GMT (11am) the longitude of the place is 15° East of Greenwich (London)
ie 15°E or +15°

(Note: If clock A shows 1 pm, the longitude of the place is -15° or 15°W)

Ex:2 The clock A set to GMT shows 4:17 AM. What is longitude?

The location is 12 NOON - 4:17 AM
= 7 hrs 43 min ahead. (+)

∴ 7 hrs ⇒ $7 \times 15^\circ = 105^\circ$

43 min ⇒ $\frac{43}{60} \times 15 = 10.75^\circ$

$+115.75^\circ$ (E)

Ex:3 The longitude of Marulan is $+150^\circ$ (E). What is the GMT at local Noon at Marulan.

Marulan is $\frac{150^\circ}{15^\circ} = 10$ hrs ahead

GMT ⇒ 12 NOON - 10 hrs.

= 2 AM

Home Work

A sailor has reached land on 8th Jun after sailing west from England. At local Noon, the ship's clock shows 5:06 pm (GMT) and the sun's elevation angle is measured as 75.6 degrees. What is the latitude and longitude of the location? Which place he has reached?