U3A Maths

14-Oct-2025 — Term 4 / Week 1

Solar Time / Local Time

Review

- Sun's elevation angle (e) at Midday or Solar Noon

  $\sin(e) = \cos(\phi - d)$

  where ϕ – Latitude

  d – Earth's declension Angle

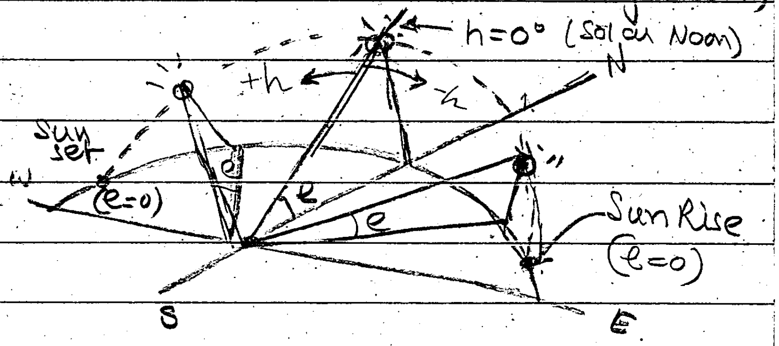
  $= 23.45^\circ \sin\left(360 \times \frac{(284+N)}{365.25}\right)$

- A (more) general equation for Sun's elevation angle (e) for any given time of the day

is as below:

$$\sin(e) = \sin(\phi).\sin(d) + \cos(\phi).\cos(d).\cos(h) \quad \text{– Eqn (1)}$$

where h – Solar Hour Angle (SHA)



- Solar Hour Angle (h) can be used to calculate Solar Time relative to Solar Noon
- For $h=0^\circ \Rightarrow$ Solar Noon (12 Noon)
- $h>0^\circ \Rightarrow$ After Noon

  $h<0^\circ \Rightarrow$ Before Noon (Morning)

- Hence, Solar Time (ST)

$$ST = 12 + h^\circ/15^\circ \quad \text{– Eqn (2)}$$

Note: Sun's movement (earth's rotation) corresponds to 24 hours.

$\therefore$ $360^\circ/24\text{ hrs} = 15^\circ/\text{hour}$

- The value of 'h' corresponds to earth's rotation or Sun's movement.

- For sun rise & Sun set, the Solar hour angle (h_s) corresponds to Sun's elevation angle $(e) = 0$.

  ie, $\sin(e) = \sin(0^\circ) = 0$

  $= \sin(\phi).\sin(d) + \cos(\phi)\cos(d)\cos(h_s)$

$\therefore$ Sun rise / Sunset

$$h_s = -\tan(\phi).\tan(d) \quad \text{– Eqn (3)}$$

- Finally:

  Sun rise (solar) Time

  $ST_{(SR)} = 12 - h_s/15$

  Sunset (solar) Time

  $ST_{(SS)} = 12 + h_s/15$

  – Eqn (4)

- To obtain the clock Time, the Solar Time needs be corrected for
  - Eccentricity of Earth's orbit
  - {22.1° to 24.5°} Wobble of earth's declension
  - Local longitude (if different from Local Standard Time longitude)

Home Work

Calculate the solar hour angle and solar time for sunrise and sunset for Kansas City (USA) on Feb 15th.

Latitude of Kansas City

$\phi = +39^\circ.3$

For 15th Feb

$N = 31 + 15 = 46$

∴ Earth's declension Angle

$$d = 23^\circ.45 \sin\left(360 \times \frac{(284+46)}{365.25}\right)$$

$$= -13^\circ.364$$

Solar Hour Angle for Sun Rise/Sunset

$$\cos(h_s) = -\tan(\phi).\tan(d)$$

$$= -\tan(39^\circ.3)\tan(-13^\circ.364)$$

$$= 0.1945$$

$$\therefore h_s = \cos^{-1}(0.1945) = 78^\circ.79$$

∴ Solar time for Sun rise (ST_{SR})

$$= 12 - h_s/15$$

$$= 12 - 78.79/15$$

$$\cong 6.75 \text{ hrs} \quad \text{or} \quad 6:45 \text{ AM}$$

Solar time for Sunset (ST_{SS})

$$= 12 + h_s/15 = 12 + 78.79/15 \cong 17.25$$

or 5:15 PM

- We can calculate the local time (LT) from the Solar Time (ST) using the following correction factor (TC)

$$LT = ST - TC$$ (in min)

where $TC = 4(\lambda_{loc} - \lambda_{LST}) + EOT$

λ_{loc} = Local longitude

λ_{LST} = Longitude of Local Std Time

EOT ⇒ Equation of Time

$$= \underbrace{9.87 \sin(2B)}_{\substack{\text{Eccentricity} \\ \text{(Elliptic path)}}} - \underbrace{7.67 \sin(B + 78^\circ.7)}_{\substack{\text{obliquity} \\ \text{(Declination } 22^\circ.1 \text{ to } 24^\circ.5)}}$$

where $B = 360 \times (N-81)/365$

EOT Eqn from Aust. Govt Space Services!

Ex: Solar Time for Sunset at Ingleburn (NSW) was calculated as 6:02 PM on 23rd Sep 25. (N=266)
Calculate the local time for Sunset

[Refer to Notes of 23rd Sep 25]

We have $\lambda_{loc} = 150^\circ.8630$ (E)

$\lambda_{LST} = 150^\circ$ (E) [Meridian]

$$B = 360 \times \frac{(N-81)}{365} = 360\frac{(266-81)}{365} = 182^\circ.47$$

$$EOT = 9.87 \sin(2 \times 182^\circ.47) - 7.67 \sin(182^\circ.47 + 78^\circ.7)$$

$$= 8.43 \text{ min}$$

∴ Time Correction

$$TC = 4(150.863 - 150) + 8.43$$

$$= 3.452 + 8.43 = 11.88 \text{ min}$$

(Say 12 mins)

Local time (LT) = ST − TC = 6:02 − 0:12 = 5:50 PM

Note: The sunset time is 5:53 PM as per "time and date.com"

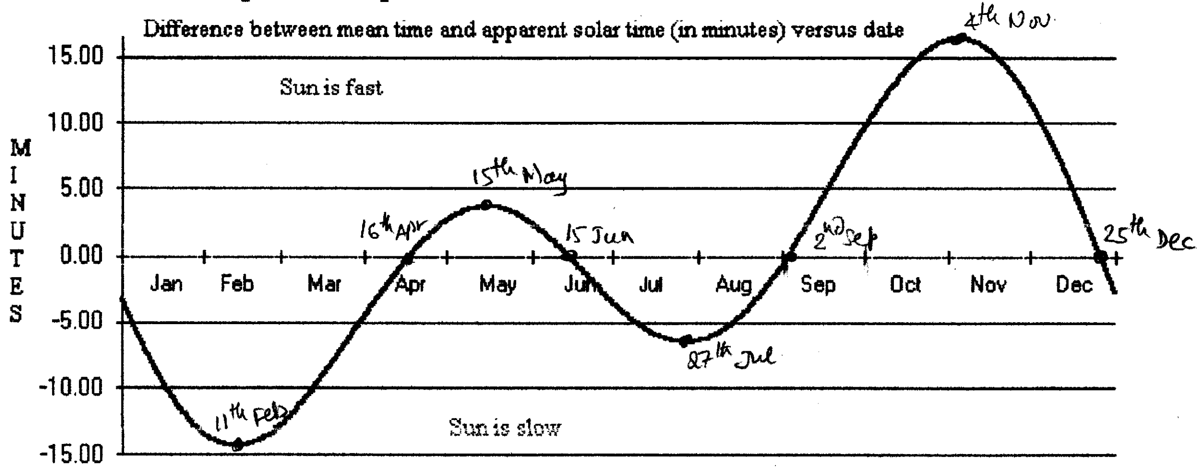
Australian Government
IPS Radio and Space Services

THE EQUATION OF TIME

The time on a sundial is not the same as the time you read from a watch. There are two reasons for this difference. One is due to our use of zonal times that are taken from fixed longitudes 15° apart. Australian Eastern Standard Time (AEST) is referred to a longitude of 150° east of the prime meridian, and this gives a time which is 10 hours ahead of Universal Time *(excluding the effect of change in local time due to day light saving)*. Sydney *(longitude 151.18°)* is quite close to longitude 150°, and so when the Sun in Sydney reaches it's highest point (e.g. on 10 July 2010) in the sky quite close to noon on a clock, in Melbourne the Sun would reach it's highest point around 12.25 pm, Melbourne being about 6.2 ° west of Sydney in longitude.

There is a second effect which displaces the actual time of solar central meridian passage or CMP from the times listed above. This effect is called the "Equation of Time", and it is the time in minutes by which the time of CMP is displaced from mean solar noon (clock time).

Graph of the Equation of Time



The equation of time results from: 1) the 23° tilt of the Earth's rotational axis, 2) the fact that the Earth moves around the Sun in an elliptical orbit. When a planet moves in an elliptical orbit, it moves faster when it is at perihelion than when it is at aphelion. This appears to surface inhabitants as if the Sun moves around the Earth faster at perihelion (January) and slower at aphelion (July). The tilt of the Earth's axis makes the Sun appear to cross one's central meridian at different times than if the axis were perpendicular to the orbital plane. These two effects add to produce the equation of time as shown above.

When the sun is "fast", CMP will occur earlier. When it is "slow", CMP will occur later. So in Melbourne *(excluding the effect of change in local time due to day light saving)*, apparent solar noon will occur around 12:04 AEST in the first week of November, and around 12:34 in the middle of February.

The actual equation of time, which is mathematically defined as apparent solar time - mean solar time, varies slightly from year to year due to variations in the Earth's eccentricity and obliquity and in the time of the solstices and equinoxes. But, for a century either side of the year 2000 it may be approximated (to an accuracy of better than one percent) by the formula:

$$EoT(minutes) = 9.87 \sin(2B^\circ) - 7.67 \sin(B^\circ + 78.7^\circ)$$

where $B^\circ = 360 (N-81) / 365$

and N is the day of the year (eg N=1 on Jan 1 and N=33 on Feb 2)

The equation of time affects any thing to do with the sun, such as sunrise and sunset times as well as the CMP time.

EOT Table

Date	Minutes
Jan 1	−3.2
16	−9.6
Feb 1	−13.5
Feb 16	−14.2
Mar 1	−12.6
16	−9.0
Apr 1	−4.2
16	0
May 1	+2.8
May 16	+3.7
Jun 1	+2.4
16	−0.4
Jul 1	−3.5
16	−5.9
Aug 1	−6.3
Aug 16	−4.4
Sep 1	−0.2
16	+4.8
Oct 1	+10.0
16	+14.2
Nov 1	+16.3
Nov 16	+15.3
Dec 1	+11.3
16	+4.8