U3A Maths

21-Oct-2025 Term 4 / Week 2

Equation of Time (EoT)

Brief Review

Sun's elevation angle (e)

$$\sin(e) = \sin(\phi)\sin(d) + \cos(\phi)\cdot\cos(d)\cos(h) \quad \text{--- Eqn (1)}$$

where ϕ – Latitude

d – Earth's declension

$$= 23.45 \sin\left(\frac{360 \times (284+N)}{365.25}\right)$$

N – Day No. of the year.

h – Solar Hour Angle (SHA)

$h = 0$ for Solar Noon / Midday

(Also, $e = 0$ for Sunrise/Sunset)

Solar Time (ST)

$$ST = 12 \pm h^\circ/15^\circ \text{ hours} \quad \text{--- Eqn (2)}$$

– "h" is obtained by measuring 'e'

– Earth's / Sun's rotation $\Rightarrow 360^\circ/24\text{ hrs} = 15^\circ/\text{hr}$

Sunrise / Sunset Solar Hour Angle (h_s)

By setting $e = 0$ in Eqn. (1)

$$\cos(h_s) = -\tan(\phi)\cdot\tan(d) \quad \text{--- Eqn (3)}$$

Sunrise (ST_{SR}) $= 12 - h_s/15$

Sunset (ST_{SS}) $= 12 + h_s/15$

Local Time

Local Time (LT) = Solar Time (ST) – Time Correction (TC)

where $TC = 4(\lambda_{Loc} - \lambda_{LST}) + EoT$

λ_{Loc} – Longitude of the Location

λ_{LST} – Longitude of Local Std. Time

EoT – Equation of Time

Equation of Time (EoT)

$$EoT = 9.87\sin(2B) - 7.67\sin(B + 78.7)$$

where $B = 360 \times (N-81)/365$

Home Work

- Calculate the local time of Sunrise and Sunset at Penrith, NSW on 21st Oct 2025.
- Check your results from "time and date.com" web site.

We have:

Penrith Post Off.: Latitude (ϕ) $= -33.7547$

Longitude (λ) $= +150.7020$

21st Oct $\Rightarrow N = 294$

$\therefore$ Earth's Declension (d)

$$= 23.45 \sin\left(360 \times \frac{(284+294)}{365.25}\right)$$

$$= 23.45 \sin(569.69) = -11.615$$

Sunrise / Sunset Solar hour Angle (h_s)

$$= \cos^{-1}[-\tan(\phi)\cdot\tan(d)]$$

$$= \cos^{-1}[-\tan(-30.7547)\cdot\tan(-11.615)]$$

$$= \cos^{-1}[-0.1373] = 97.8916$$

Solar Time for Sunset (ST_{SS})

$$= 12 + h_s/15$$

$$= 12 + 97.8916/15^\circ = 12 + 6.526$$

$$= 18.526 \text{ hrs} \Rightarrow 18:31.56 \quad (\text{hrs : min})$$

Solar Time for Sunrise (ST_{SR})

$$= 12 - h_s/15$$

$$= 12 - 97.8916/15^\circ$$

$$= 12 - 6.526 = 5.474 \text{ hrs.}$$

$$\Rightarrow 5:28.44 \quad (\text{hr : min})$$

Time Correction (TC) [min]

$\lambda_{LST} = 150°$ — for Aust. Eastern Std. Time

$TC = 4(\lambda_{local} - \lambda_{LST}) + EoT$

$= 4(150.7020 - 150) + EoT$

$= 2.808 + EoT$

Calculate EOT

$B = \frac{360 \times (N-81)}{365} = \frac{360(294-81)}{365}$

$= 210°.08$

$EOT = 9.87 \sin(2B) - 7.67 \sin(B + 78°.7)$

$= 9.87 \sin(2 \times 210°.08) - 7.67 \sin(210°.08 + 78°.7)$

$= 15.823$ min

$\therefore TC = 2.808 + 15.823 = 18.63$ min

Local Time for Sunset (LT_{SS})

$LT_{SS} = ST_{SS} - TC$

$= 18:31.56 - 18.63$ (hr : min − min)

$\cong 18:13$ (hr : min)

$\therefore$ Add 1 hr Daylight Saving

$\therefore$ Sunset $\Rightarrow$ 19:13 (hr min)

As per "timeanddate.com" $\Rightarrow$ 19:14 (hr min) ✓

Local Time for Sunrise (LT_{SR})

$LT_{SR} = ST_{SR} - TC$

$= 5:28.44 - 18.63 =$ (hr : min − min)

$= 5:9.81 + 1\text{hr} \cong 6:10$ (hr min)

As per "timeanddate.com" $\Rightarrow$ 6:08 (hr min) ✓

Equation of Time

- The Solar Time is determined by the position of the "Apparent" Sun as seen at the local location on earth. This is the time provided by the Sundial. This is, in fact, the correct local time.

- Hence, the solar time takes into account the following:
  - Earth's elliptical orbit and the eccentricity of the orbit. This causes earth to move fastest at perihelion (Closest to Sun in January)

and slowest at aphelion (farthest from Sun in July). This causes the motion of the "Apparent" Sun speed up or slow down depending on the day of the year.

- Tilt of the Earth's axis (obliquity) which affects the path of the apparent Sun throughout the year (causing the earth's seasons)

- The clock time ignores the above effects! Hence, clock time assumes

a fictitious Sun (called the "Mean Sun") that moves along the celestial equator at a uniform rate. However, the clock time is designed so that the Earth orbits the Sun in exactly one year, hence, ensuring that the actual Sun (Apparent Sun!) and the "Mean Sun" are Synchronised on an yearly basis!

- Equation of Time is precisely defined as

EoT = GHA(Apparent Sun) − GHA(Mean Sun)

where

GHA – Hour Angle at Greenwich

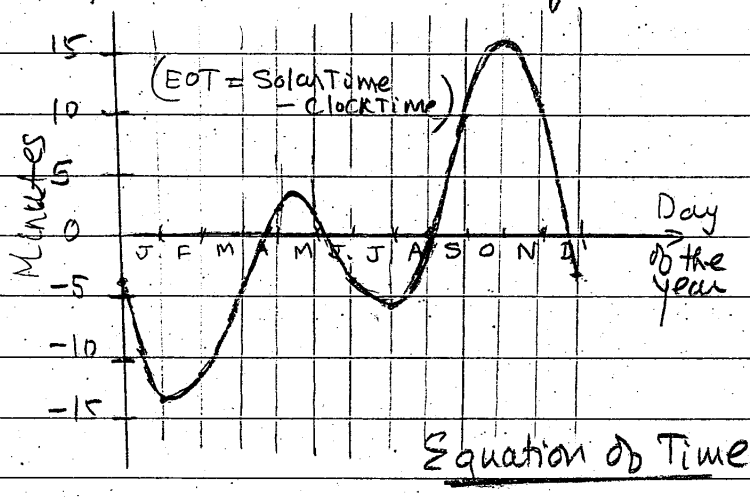
- However we can use the simpler definition,

EoT = Solar Time − Clock Time

at Greenwich.

- The EoT can be determined by various methods. For Ex.
  (a) by precise modelling of earth's motion around the Sun
  (b) by noting down the

"clock time" say at Solar Noon every day of the year and tabulating the difference. We can then develop an equation for such a table of values.



Equation of Time

- Fourier Transform of the above graph, shows that it consists predominently of two Sine waves,

one with a frequency of 1 year (365 days) and another with double the frequency (½ year)

- Such an analysis provides us the following Equation

$$EoT = 9.87 \sin(2B) - 7.67 \sin(B + 78^\circ.7)$$

where $B = 360 \times \frac{(N-81)}{365}$ $\begin{cases} N=1 \Rightarrow \text{Jan 01} \\ N=365 \Rightarrow \text{Dec 31} \end{cases}$

- The above two sinusoids can be plotted separately and "summed" to obtain the "Equation of Time" graph!