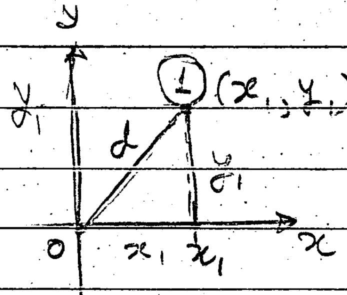
(Week 3 – No class)

U3A Maths

4-Nov-2025 Term 4 / Week4

Distance between 2 Points

- Our objective is to find distance between two points on earth (considered as a perfect sphere!)
- However, let us start with the basic algebra for two dimensions and other relevant formulae
- Let us consider a simple case in two dimensions (plane surface).



- Distance between a given point at ① located at (x_1, y_1) and the origin (0,0)

- Using Pythagoras theorem, we have

$$d^2 = (x_1^2 + y_1^2) \text{ or } d = \sqrt{x_1^2 + y_1^2}$$

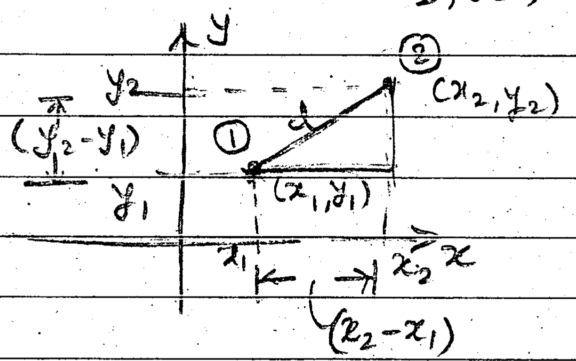
Ex(1) $x_1 = 3$, $y_1 = 4$

$$d = \sqrt{3^2 + 4^2} = \sqrt{25} = \underline{5}$$

- Let us now find the distance between two random points,

① – located at (x_1, y_1)

& ② – located at (x_2, y_2)



- Using the Pythagoras theorem again

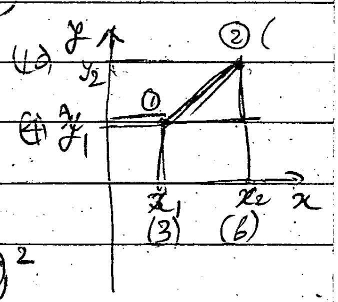
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

The above equation is quite general & works for any 2 points.

Ex(2) Calculate the distance between points

① ⇒ $(x_1, y_1) = (3, 4)$

② ⇒ $(x_2, y_2) = (6, 10)$



we have,

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$= (6 - 3)^2 + (10 - 6)^2$$

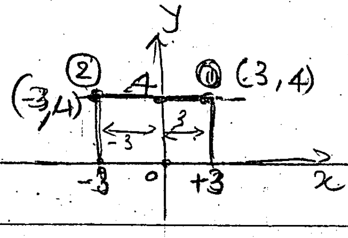
$$= 3^2 + 4^2 = 25$$

$$\therefore d = \sqrt{25} = 5!$$

Ex.3 Calculate distance between points

① ⇒ $(x_1, y_1) = (3, 4)$

② ⇒ $(x_2, y_2) = (-3, 4)$



- From the figure, the distance is obviously 6 !
- Let us use our equation,

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

we have

$$x_1 = 3, \; y_1 = 4$$

$$x_2 = -3, \; y_2 = 4$$

$$\therefore d^2 = (-3-(3))^2 + (4-4)^2$$

$$= (-6)^2 = 36$$

$$\therefore d = \sqrt{36} = \underline{\underline{6}}$$

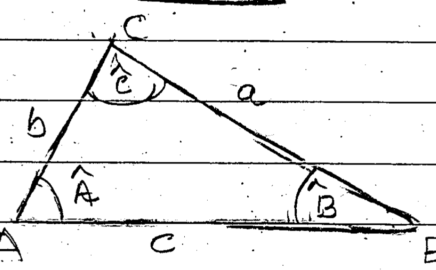
<u>Home work</u>

- Find the distance between
  ① $\Rightarrow (x_1, y_1) = (3, 4)$
  ② $\Rightarrow (x_2, y_2) = (-3, -4)$

- While working with a sphere, it is useful (necessary!) to use a generalised Pythagoras theorem, which is applicable to any given triangle - not just for right angled triangle.
- Such a formula is called the "<u>Law of Cosines</u>"
- "Law of Cosines" has its origin in Euclid's geometric work in 300 BCE Greece! However, the first explicit statement

was done by Persian mathematician Jamshid-Al-Kashi in 15th century!! This was later adapted by the Western world in the 16th century by François Viète (French).

- <u>Law of Cosines</u>



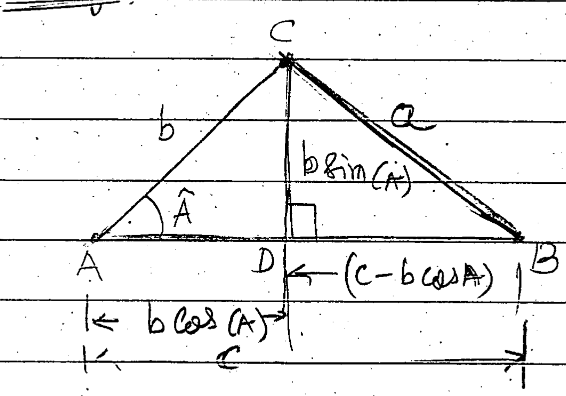
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

<u>Note</u> we can also write

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

& $$b^2 = c^2 + a^2 - 2ca\cos(B)$$

<u>Proof</u>



Considering the right angled triangle CDB

$$a^2 = [b\sin(A)]^2 + [c - b\cos(A)]^2$$

$$= b^2\sin^2(A) + c^2 + b^2\cos^2(A) - 2bc\cos(A)$$

$$\therefore a^2 = b^2(\sin^2(A) + \cos^2(A)) + c^2 - 2bc\cos(A)$$

$$\boxed{a^2 = b^2 + c^2 - 2bc\cos(A)}$$