U3A Maths

11-Nov-2025 Term 4 / Week5

Distance between 2 Points on sphere

Review two Points

- Distance between (x_1, y_1) & (x_2, y_2) on a 2 dimensional plane

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is based on Pythagoras theorem for right angled triangle.

- We also noted that a more general form of the Pythagoras applicable to any triangle is the "Law of Cosines"

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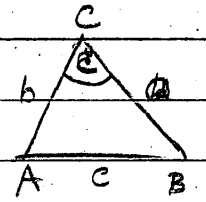
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$



- This was developed by Euclid in about 300 BC. (Pythagoras ⇒ 500 BC)

Ex:1 Calculate the distance between $(4, 3)$ and $(9, 4.5)$
(x_1, y_1) (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(9-4)^2 + (4.5-3)^2}$$

$$= 5.2202$$

Note that, we can also write,

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

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$$d = \sqrt{(4-9)^2 + (3-4.5)^2}$$

$$= 5.2202$$

- The above equation can be logically extended to 3 or more dimensions!

- Distance betwen (x_1, y_1, z_1) & (x_2, y_2, z_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Ex2 Calculate the distance between $P_1 \Rightarrow (0,0,0)$ & $P_2 \Rightarrow (1,1,1)$

$$d = \sqrt{(1-0)^2 + (1-0)^2 + (1-0)^2}$$

$$= \sqrt{1+1+1} = \sqrt{3} = 1.732$$

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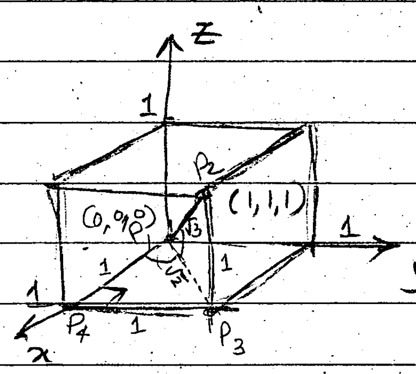
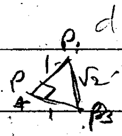
Plotting the result

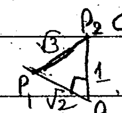


Note: 'd' is the diagonal distance in a cube! Pythagoras theorem in 3-dimensions!

- Using Pythagoras theorem



dist. P_1 to P_3 ⇒ $\sqrt{1^2 + 1^2} = \sqrt{2}$



dist P_1 to P_2 ⇒ $\sqrt{(\sqrt{2})^2 + 1^2}$

$= \sqrt{3}$

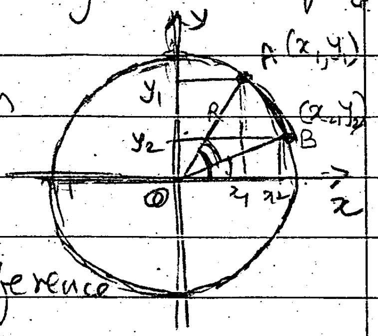
- Let us now consider the distance between two points on a circle along the circumference

- Distance between

$A \Rightarrow (x_1, y_1)$

$B \Rightarrow (x_2, y_2)$

along the circumference



Expressing in polar form:

$$A \Rightarrow OA \angle AOx$$
$$= \sqrt{(x_1^2+y_1^2)} \angle \tan^{-1}(y_1/x_1)$$
$$= R \angle \theta_1 \quad \text{(say)}$$

Similarly $B \Rightarrow R\angle\theta_2$ when $\theta_2 = \tan^{-1}(y_2/x_2)$

$\therefore$ Distance along the circumference

$$d_{AB} = R(\theta_1 - \theta_2)$$

where $\theta \Rightarrow$ is in "radians"

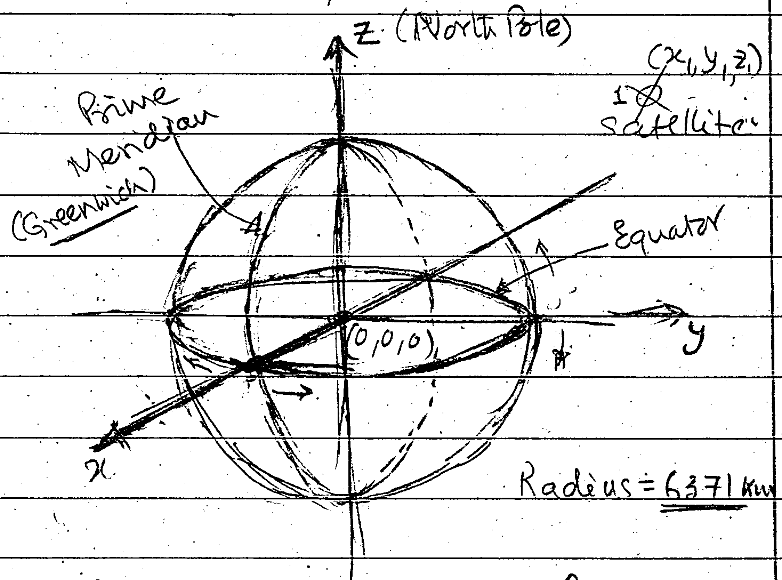
Ex 3 Find the angular distance between

$A = 5\angle 60°$ (θ_1) & $B = 5\angle 30°$ (θ_2)

$$\therefore d = 5(\underbrace{\theta_1 - \theta_2}_{\text{in radians}})$$

$$= 5(60-30)\frac{\pi}{180} = 2.618$$

- dist on unit circle = Angle in Radians!
- The above helps to calculate the distance on earth – assuming it as a perfect sphere!

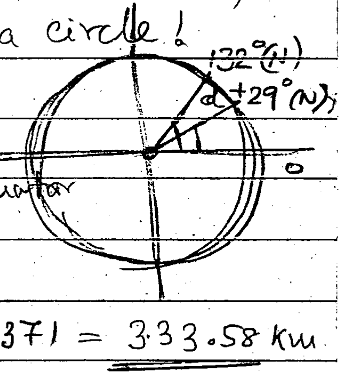


- The above reference frame is defined in WGS-84 standard (World Geodetic System -1984) and is used for Latitude & Longitude on earth and also for the location of satellites!

Ex.4 Find the anglar distance between points on earth with coordinates

$(+32°\,N, +55°\,E)$ and $(+29°\,N, +55°\,E)$

Since longitude is the same, we can consider it as a circle!



$\theta_2 - \theta_1 = 32° - 29°$
$= 3°$

$$\therefore d = 3° \times \frac{\pi}{180} \times 6371 = 333.58 \text{ km}$$

Quite often 'd' is expressed as

$d = 3° \times 60' = 180'$ (minutes)

or 180 NM (Nautical Miles)

$1 \text{ NM} \triangleq 1.852 \text{ km}$ $\therefore d = 180 \times 1.852 \triangleq 333.36 \text{ km}$

Civil Aviation Std ↙ IEEE Std. ↙

Nautical Mile (NM or nmi)

- 1 nautical mile corresponds to 1′ (minute) longitude along the equator.
- Earth's circumference at the equator = 40,000 km

∴ 40,000 km ⇒ 360° × 60′ minute

∴ 1 minute (1 NM) ⇒ $\frac{40,000}{360 \times 60} = \frac{1.85185\ldots}{\text{km}}$

1 NM ≅ 1.852 km

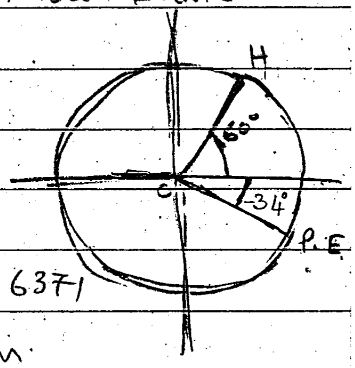
Ex. 2 Given that coordinates of Helsinki is $(+60°, +25°)$ (N, E) and

Port Elizabeth $(-34°, +25°)$ (S, E)

(a) Calculate the distance between the cities

(b) Flight time if the plane travels at 620 km/hr

Both cities are on the same longitude!



∴ Distance

$d = (60° - (-34°)) \times \pi/180 \times 6371$

$= 10,452$ km

Flight time: $t = \frac{10,452 \text{ km}}{620 \text{ km/hr}} = 16.86$ hr (16 hrs 52 m)