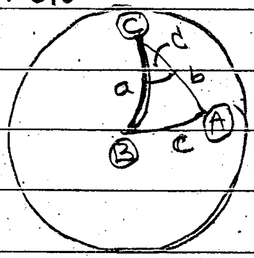
U3A Maths

25-Nov-2025 Term 4 / Week 7

Dist between Lat/Long Points

- We derived the formula for dist. bet. 2 points on a unit sphere and called it "spherical Law of Cosines", as below

$\cos(c) = \cos(a)\cdot\cos(b) + \sin(a)\sin(b)\cos(d)$



where,

c – (Radial) distance between Ⓐ & Ⓑ expressed in "Radians"

∴ Actual Distance for sphere of Radius 'R'

$d = R \cdot c$

- For the earth, we can use average radius as $R = 6,371$ km

- We chose the triangle so that node Ⓒ is the north pole and arcs Ⓑ-Ⓒ & Ⓐ-Ⓒ correspond to longitudes.

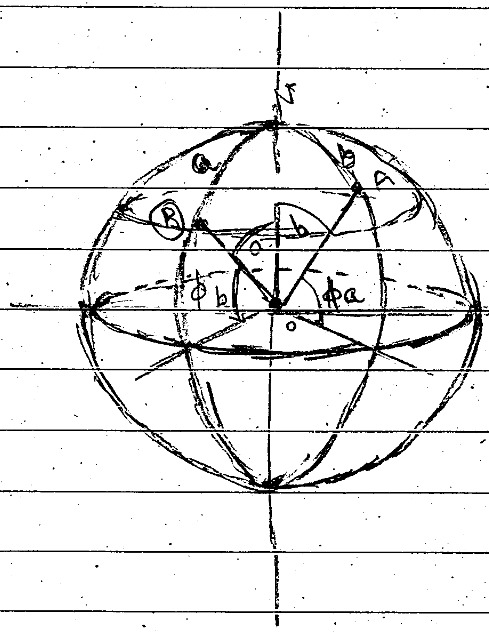
∴ Angle 'd' = Longitude Ⓑ − Longitude Ⓐ

ie, $d = \lambda_B - \lambda_A$

- The angles corresponding arcs 'a' & 'b' was based on conventional axes, with vertical axis as the reference.

- Hence, they don't (directly) correspond to latitudes.

- Let us have a second look at angles 'a' and 'b'



- From the Figure, the angle 'a' is the 'complement' of latitude angle of Ⓑ, ie, ϕ_B

$\therefore a = 90^\circ - \phi_B$

Similarly, $b = 90^\circ - \phi_a$.

Substituting the above, we get,

$\cos(c) = \cos(90^\circ - \phi_B)\cdot\cos(90^\circ - \phi_A) + \sin(90^\circ - \phi_B)\cdot\sin(90^\circ - \phi_A)\cdot\cos(d)$

We now have the final Equation:

$\cos(c) = \sin(\phi_A)\sin(\phi_B) + \cos(\phi_A)\cdot\cos(\phi_B)\cdot\cos(\lambda_A - \lambda_B)$

where ϕ_A & ϕ_B are latitude angles!

<u>Note</u>: the above formula gives the distance "as the crow flies". Hence, they are relevant for air & sea navigation.

<u>Ex.1</u> Calculate the distance between
Melbourne ⇒ $(-37.6656 \; S, +144.8419 \; E)$
& Singapore ⇒ $(+1.3519 \; N, +103.9896 \; E)$

We have, $\phi_A = -37^\circ.6656$, $\lambda_A = 144^\circ.8419$

$\phi_B = +1^\circ.3519$, $\lambda_B = 103^\circ.9896$

$$\text{Cos}(c) = \text{Sin}(-37.6658)\,\text{Sin}(+1.3519) + \text{Cos}(-33.6656).\text{Cos}(1.3519) \times \text{Cos}(144.8419 - 103.9896)$$

$$\therefore \text{Cos}(c) = 0.584418$$

$$c = \text{Cos}^{-1}(0.584418) = 54^\circ.238$$

$$\therefore d = 54^\circ.238 \times \frac{\pi}{180} \times 6371$$

$$= \underline{6031 \text{ km}}$$

[Checked by App ⇒ 6033 km]

<u>Ex.2</u> Calculate the distance between
Brisbane ⇒ $(-27.3883 \; S, +153.1261)$
Singapore ⇒ $(+1.3519 \; N, +103.9896)$
We have,

$\phi_A = -27^\circ.3883$, $\lambda_A = +153^\circ.1261$

$\phi_B = +1^\circ.3519$, $\lambda_B = +103.9896$

$$\text{Cos}(c) = \text{Sin}(-27.3883)\,\text{Sin}(1.3519) + \text{Cos}(-27.3883)\,\text{Cos}(1.3519) \times \text{Cos}(153^\circ.1261 - 103^\circ.9896)$$

$$\text{Cos}(c) = 0.5699$$

$$\therefore c = \text{Cos}^{-1}(0.5699) = 55^\circ.25619$$

$$\therefore d = 55^\circ.25619 \times \frac{\pi}{180} \times 6371$$

$$= \underline{6{,}144 \text{ km}}$$

[Checked by App ⇒ 6,144 km]

<u>Ex.3</u> Calculate the dist bet.
Sydney Airport ⇒ $(-33.9518, 151.1799)$
Penrith P.O. ⇒ $(-33.7549, 150.7019)$
We have

$\phi_A = -33.9518$, $\lambda_A = 151.1799$

$\phi_B = -33.7549$, $\lambda_B = 150.7019$.

Repeating above steps

$$\text{cos}(c) = 0.99997\ldots$$

$$\therefore c = \text{Cos}^{-1}(0.99997) = 0.44381$$

$$\therefore d = 0.44381 \times \pi/180 \times 6371$$

$$= \underline{49.34 \text{ km}}$$

[Checked by App ⇒ 49.27 km]

- For shorter distances, the "spherical cosine law" will give inaccurate results, if the calculation precision is not adequate.
- However, this is not an issue with modern calculators.
- It is an issue for manual calcs and when 4 digit Sine/Cos tables are used.

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- Let us check

$\cos(1^\circ) = 0.99984\ldots$

$\cos(0.5^\circ) = 0.99996\ldots$

$\cos(0.2^\circ) = 0.9999939\ldots$

For a 4 digit 'cos' table, we cannot distinguish between 0.5° & 0.2°

$0.5^\circ \Rightarrow d = 0.5^\circ \times \pi/180 \times 6371$

$= 55.59$ km

- Hence, the results are inaccurate for values below 0.5°.

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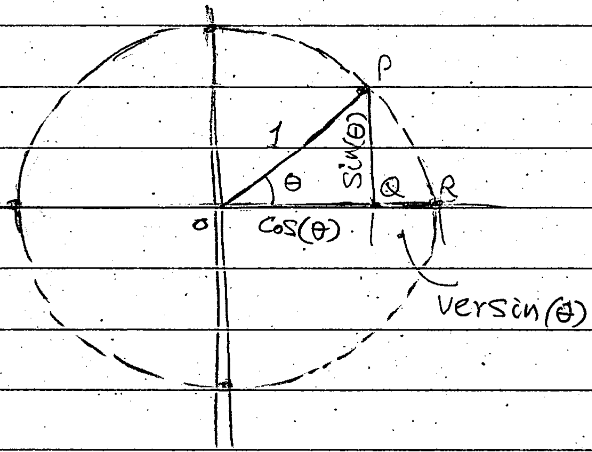
- This problem was avoided by using a modified formula, called the "Haversine" formula.

- What is Haversine?



Dist Q - R $\Rightarrow$ is called Versin(θ)

It was in popular use!

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versine $\Rightarrow$ "Versed" sine!

From the above figure,

$\text{Versin}(\theta) = 1 - \cos(\theta)$

Another associated function is,

Half of versin(θ) $\Rightarrow$

Haversine(θ) $\Rightarrow$ Hav(θ)

$\therefore \text{Hav}(\theta) = \frac{1}{2}[1 - \cos(\theta)]$

We can show that,

$[1 - \cos(\theta)] = 2\sin^2(\theta/2)$

$= \sin(\theta) \cdot \tan(\theta/2)$

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- The above identities are used to modify "spherical law of cosines" and obtain "Haversine" formula.

- Haversine formula gives more accurate results for shorter distances.

- Haversine Formula

$$x = \sin^2\left(\frac{\phi_A - \phi_B}{2}\right) + \cos(\phi_A)\cos(\phi_B) \cdot \sin^2\left(\frac{\lambda_A - \lambda_B}{2}\right)$$

$$c = 2 \times \tan^{-1}\left(\sqrt{x}/\sqrt{1-x}\right) \text{ deg}$$

Finally,

$$d = c^\circ \times \pi/180 \times 6371 \text{ km}$$