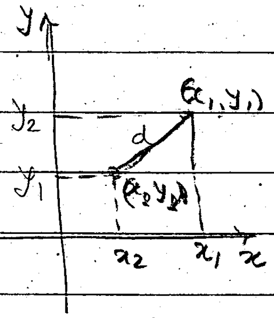
U3A Maths

2-Dec-2025 Term 4 / Week 8

Intro to GPS Location

- We know how to calculate the distance between two points in a 2-dimensional surface.



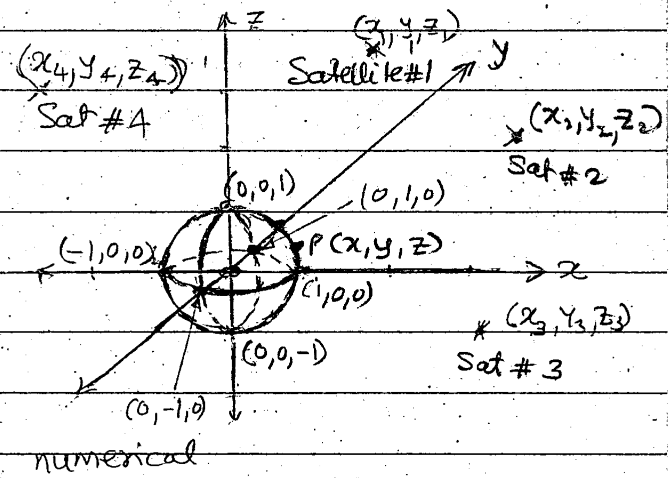
For Ex:

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

- We can also extend the above equation to 3-dimensions,

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

- For location using GPS, we need to consider
  - 3-dimensional space
  - and - spherical surface of the Earth



- For numerical calculations, it is convenient to consider the earth as a <u>unit sphere</u> and the satellite distances as multiples of earth's radius.

- The location on earth is expressed as a point in 3-dimensions.

- In the above Figure, the locations are as below:

  - Point 'p' on earth : (x, y, z)
  - Satellite #1 : (x_1, y_1, z_1)
  - Satellite #2 : (x_2, y_2, z_2)
  - Satellite #3 : (x_3, y_3, z_3)
  - Satellite #4 : (x_4, y_4, z_4)

- Note that the "Centre of Earth" is taken as the origin and has coordinates of $(0,0,0)$

- Satellites have atomic clocks and continuously transmit the time (t) and their exact location (x, y, z)

- From the above data, the <u>GPS Receiver (P)</u> on earth can calculate the distance to satellite and exact location of 'P' (x, y, z)

- <u>GPS Receiver</u> at 'P' can only receive data from satellites which are "<u>visible</u>" to it.

- To perform the calculations, <u>at least</u> 4 satellites must be visible!

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- Hence, GPS system consists of 24 satellites in polar orbits, in 6 orbital planes.
- They orbit at an altitude of approximately 20,200 Km. at a speed of about 14,000 km/h
- GPS (Global Positioning System) was developed by Dept. of Defence in the US. (1973 - Defence Use 1980's - Public Access).
- Other similar systems are operated by:
  - E.U. ⇒ Galileo
  - Russia ⇒ GLONASS
  - China, India, Japan etc.

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- Before jumping to the GPS system, let us understand the concept by considering a 2-dimensional system.

Problem Definition

- The basic problem is to establish the location of 'P' (x & y coordinates) by knowing the location of another point, say $A(x_1, y_1)$ and distance (d_1) to P.
- In a 1-dimensional the system, the solution is trivial!

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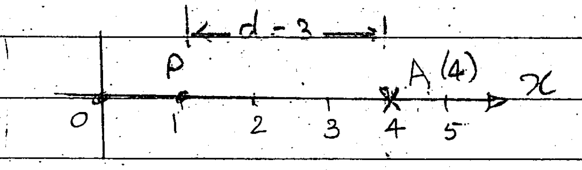

∴ Location of A ⇒ (4)
Distance (d) to P = 3

∴ Location of P = 4 − 3 = 1 !

- The problem is a more complicated in 2 dimensions.
- Let us say we are given that,
  Location of $A(x_1, y_1) = (14, 45)$
  distance to P ⇒ $d_1 = 39$

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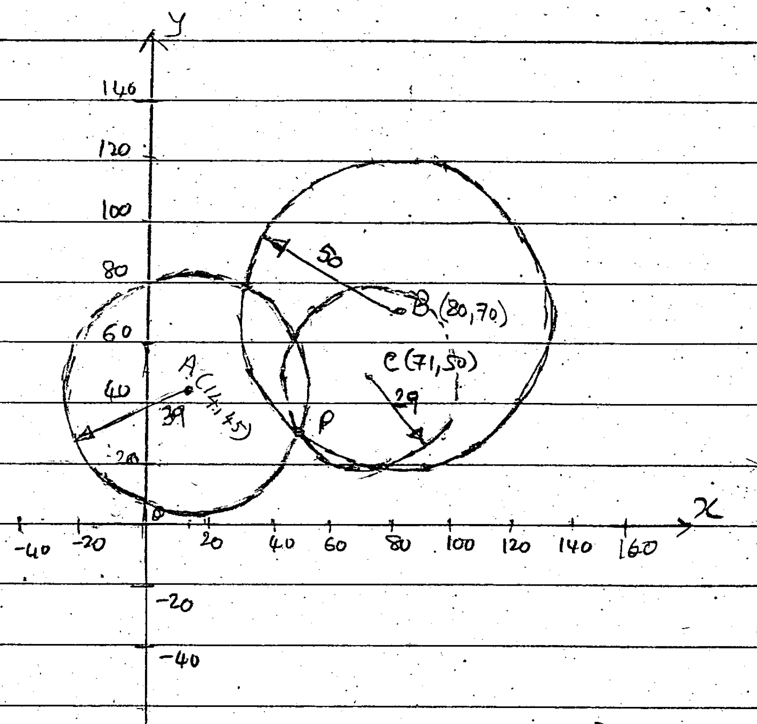
Mathematically, we have

$$(x - x_1)^2 + (y - y_1)^2 = d_1^2$$

ie $(x - 14)^2 + (y - 45)^2 = 39^2$ — (1)

$$\therefore (x^2 - 28x + 196) + (y^2 - 90y + 2{,}025) = 1521$$

- Obviously, we do not have sufficient information to find location of 'P'.
- In fact, we have infinite solutions. For example, if we assume $x = 0, 1, 2 \ldots$ then we get a quadratic equation giving 2 values for 'y'
- Let us visualise the probem graphically!



- As shown in the above Figure, the Eqn ① is a circle. Hence, point 'P' can be located at any point on the circumference!

- Hence, we need more equations or more data.
- Let us say, we are given another point 'B' located at $(x_2, y_2) \Rightarrow (80, 70)$ at a distance 'd_2' $= 50$ from 'P'.
- ∴ We have

$$(x-x_2)^2 + (y-y_2)^2 = d_2^2$$

$$(x-80)^2 + (y-70)^2 = 50^2 \quad — ②$$

- We now have 2 equations with 2 unknowns, but

they are non-linear (double quadratic!) equations.

- We can solve it with some difficulty. However, the Figure shows that 'P' can be located at two positions, corresponding to the intersection of the two circles, with centres at 'A' & 'B'.
- Therefore, we still need more data!

- Let us say, we are given one more point 'C' located at $(x_3, y_3) \Rightarrow (71, 50)$ at a distance $(d_3) = 29$ from P.

∴ We have

$$(x-x_3)^2 + (y-y_3)^2 = d_3^2$$

$$(x-71)^2 + (y-50)^2 = 29^2 \quad — ③$$

- We can plot the circle corresponding to Eqn ③. This gives us the location of of Point 'P' $\Rightarrow (x, y)$ is $(50, 30)$??
- How can we solve the above equations algebraically?
- The equations in the final form are as below:

$$x^2 + y^2 - 28x - 90y + 700 = 0 \quad — ①$$

$$x^2 + y^2 - 160x - 140y + 8{,}800 = 0 \quad — ②$$

$$x^2 + y^2 - 142x - 100y + 6{,}700 = 0 \quad — ③$$

Home work!