U3A Maths

9-Dec-2025 Term 4 / Week 9

Location Using GPS.

Home Work

Solve The following Eqns to find The location (x, y).

$$x^2+y^2-28x-90y+700=0 \quad -(1)$$
$$x^2+y^2-160x-140y+8{,}800=0 \quad -(2)$$
$$x^2+y^2-142x-100y+6{,}700=0 \quad -(3)$$

Subtracting, ie, (1) - (2) & (2) - (3):

$$132x+50y=8100$$
$$114x+10y=6{,}000$$

Solving The equations, we get

$$x=50$$
$$y=30$$

Hence, the location of point $P(x,y) = (50, 30)$

Ex.2 only the data for points (A) & (B) are as given (as in the previous problem). However, it is given that the point 'p' is located on a circle of radius of 58.3095 with the centre at (0,0). Find the position of point 'p'.

∴ We have, as before,

$$x^2+y^2-28x-90y+700=0 \quad -(1)$$
$$x^2+y^2-160x-140y+8{,}800=0 \quad -(2)$$

We have also been given that 'p' lies on the circle with the centre at (0,0) or the origin. ∴ We have,

$$(x-0)^2+(y-0)^2=(58.3095)^2$$

ie, $x^2+y^2-3400=0 \quad -(3)$

As before, Subtracting

$$(1)-(2) \Rightarrow \quad 132x+50y=8100$$
$$(2)-(3) \Rightarrow \quad 160x+140y=12{,}200$$

Solving the equations, we get

$$x=50$$
$$\& \quad y=30$$

Effect of Theory of Relativity

- As per the Special theory of Relativity, the clock (time) on a moving object slows down, compared with the clock at the stationery object.

- Satellites have atomic clocks which are extremely accurate, However, they do slow down relative to stationery object, as per Einstein's theory.

Hence, they need to be corrected!

- Let us have a closer look.

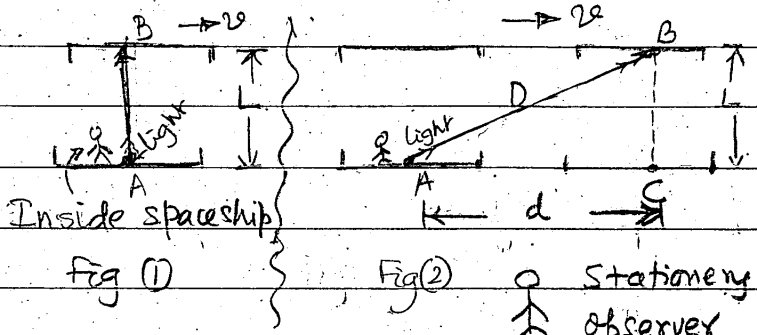


Fig (1) Fig (2)

- In Fig (1) time taken for the light from (A) to (B) is

$\Delta t_s = L/c$ —(1) [time = $\frac{\text{Dist}}{\text{speed}}$]

where 'c' the speed of light

(Note that the speed of the spaceship 'v' has

no effect on the Equation)

- In Fig (2), for a stationery observer, by the time the light touches the ceiling (B), the space-ship has moved a distance of 'd'

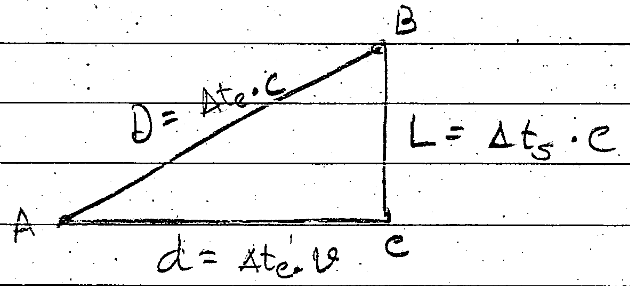
- For a stationery observer, the time taken for the light to travel from A to B (floor to ceiling of the spaceship)

$\Delta t_e = D/c$ —(2)

Also, the distance travelled by the spaceship during time "Δt_e"

$d = \Delta t_e \times v$ —(3)

We can now sketch as below:



Using Pythagoras theorem

$$D^2 = L^2 + d^2$$

or $(\Delta t_e)^2 c^2 = (\Delta t_s)^2 \cdot c^2 + (\Delta t_e)^2 v^2$

Simplifying, we get,

$$\Delta t_e = \Delta t_s \left(\frac{1}{\sqrt{1 - v^2/c^2}} \right)$$

- To get a better feel, let us say the spaceship is travelling at $v = 0.9c$ (or $v/c = 0.9$)

∴ We get:

$$\Delta t_e = \Delta t_s \cdot \frac{1}{\sqrt{(1 - 0.9^2)}}$$

$$\Delta t_e = \Delta t_s \times 2.29$$

Stationery clock — Clock on Spaceship

ie, clock on earth moves 2.29 times faster than the clock on the spaceship!

(The above applies to both mechanical & biological clocks!)

- Considering the GPS system,

speed of satellite (v) $\cong$ 14,000 km/h (3.9 km/s)

speed of light (c) = 299,792 km/s

for one day in the satellite

$\Delta t_s = 24 \times 60 \times 60 = \underline{86,400}$ sec

$$\therefore \Delta t_e = \frac{86,400}{\sqrt{1-(3.9/299,792)^2}}$$

$\cong$ 86,400.000,0073 seconds

$\therefore$ the clock on earth moves approx. 7 μs / day faster than the satellite clock.

or satellite clock loses 7 μs / day

- As per "General theory of Relativity", the gravity (space-time warp) also affects the time. The time 'Δt_r' at a dist 'r' from gravitational centre

$$\Delta t_r = \Delta t_0 \sqrt{1 - \frac{2GM}{rc^2}}$$

where, M – Mass of gravitational object (Earth)

G – Universal Gravitational Constant

c – speed of light (m/s)

r – dist. from gravitational centre

Δt_0 – time unaffected by gravity

Note: we have, r_{earth} = 6,371,000 m

$r_{satellite}$ = 26,371,000 m

(Dist from centre of Earth)

Using $G = 6.67408 \times 10^{-11}$

$M = 5.972 \times 10^{24}$ kg

$c = 2.998 \times 10^{8}$ m/s

Using Δt_0 = 24 hrs. (86400 sec)

Earth $\Rightarrow \Delta t_e$ = 86,399.999940... sec

Satellite $\Rightarrow \Delta t_s$ = 86,399.999985... sec

The clock on earth moves 45 μs slower than the satellite

or satellite clock gains 45 μs / day

$\therefore$ Total effect is, satellite clock runs faster by 45 − 7 = 38 μs per day

- Finally, 38 μs corresponds to a distance measurement error of $\left(\frac{38 \times 10^{-6}}{(s)} \times 299,792 \text{ km/s}\right)$

= 11.4 km / day

- Hence, atomic clock on satellites needs be corrected by 38 μs everyday.

- This correction is built into satellite clocks!!

- GPS is the most useful practical application of Einstein's theory of Relativity.