U3A Maths

3-Feb-2026 Term 1 / Week 2

Time Dilation
(Special Theory of Relativity)

- This topic was covered in last term as a part of the topic on "Global Positioning System" (GPS).

- In fact, this is considered to be one of the first and the prominent application of "Einstein's Special Relativity" for daily use!

- Note that Einstein was awarded Nobel prize in 1921 for "Photo Electricity" because the committe thought that Relativity theory had no practical application!!

1905 ⇒ 3 papers by Einstein:
- Photo electricity
- Brownian Motion
- Special Relativity ($E=mc^2$)

1915 ⇒ General theory of Relativity to incorporate Gravitation.

- For our purposes, this topic will also serve as an introduction to Algebra!

- By definition,

$$\text{Speed} = \frac{\text{Distance (travelled)}}{\text{Time (taken)}}$$

$$S = \frac{D}{T}$$

The above equation is used for "Average" speed.

- More commonly, the term velocity (v) is used, which represents "Instantaneous" speed, and often associated with a direction.
- Considering small distances, the equation is written as:

$$v = \frac{\Delta s}{\Delta t} \text{ or } \frac{ds}{dt}$$

The term 's' is commonly used for distance,

as the equation was derived for microscopic particles moving in a fluid (Liquid / gas) [Eintein derived the relavant equations for Brownian Motion]

Note: 's' is used for "displacement" (distance), since 'd' is used in (d/dt)!!

Time Dilation

- As per the special theory of Relativity, the clock (time) on a moving object slows down (including the biological clock) relative to the clock on the stationary object!

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- Let us explore the following illustration, when a light source (A) switched 'ON".

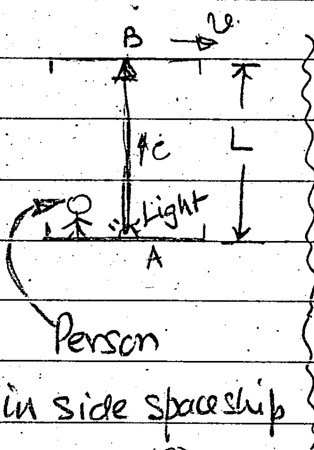

Fig ①

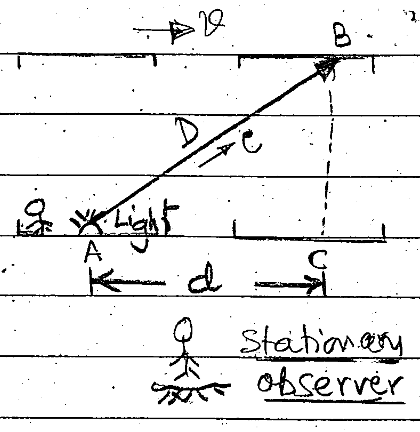

Fig (2)

- In Fig ①, the time taken for light to travel from Ⓐ to Ⓑ for an observer on the spaceship is:

$$\Delta t_s = L/c \quad -①$$

where L - height, c - velocity of light

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- Note that the speed of the spaceship (v) has no effect on the equation.

- In Fig (2), for a stationary observer outside the spaceship, by the time the light touches the ceiling (B), the spaceship has moved a distance of 'd'

- For Stationery observer, the time taken for the light to travel from Ⓐ to Ⓑ (i.e. from floor to the ceiling of the spaceship) is:

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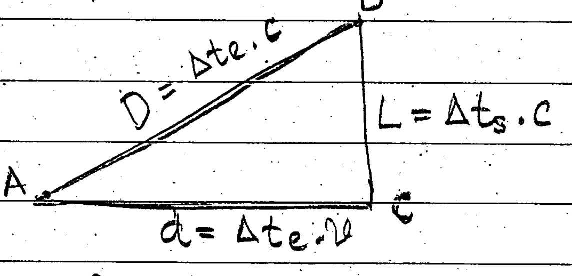
$$\Delta t_e = \frac{D}{c} \quad - ②$$

- Also, the distance travelled by the spaceship during the time interval "Δt_e" is:

$$d = v \times \Delta t_e \quad - ③$$

- We can now sketch as below:



Using Pythagoras theorem, we have,

$$D^2 = L^2 + d^2$$

(Proof of Pythagoras theorem?)

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∴ We have,

$$(\Delta t_e)^2 . c^2 = (\Delta t_s)^2 c^2 + (\Delta t_e)^2 v^2$$

- Simplifying, we get,

$$\boxed{\Delta t_e = \Delta t_s \left(\frac{1}{\sqrt{1 - v^2/c^2}} \right)}$$

Time Dilation Equation!

Recall that:

Δt_e - time elapsed on the Stationary clock.

Δt_s - time elapsed on the Space-ship clock.

Ex.1 Calculate the relative time elapsed if the spaceship is travelling at a speed of 90% of the velocity of light.

We have: $v = 0.9c$

As per Time Dilation Eqn

Stationary clock — spaceship clock.

$$\Delta t_e = \Delta t_s \times \frac{1}{\sqrt{1-\left(\frac{0.9c}{c}\right)^2}}$$

$$= \Delta t_s \times \frac{1}{\sqrt{1-0.9^2}} = \Delta t_s \times 2.29$$

- In other words, the clock on earth moves 2.29 times faster than the clock on the spaceship
- This applies for biological clock also. Hence, a person born & living on earth is 2.29 times older than the person on the spaceship.

For ex:

Person on space ship ⇒ 50 years

Person on earth ⇒ 50 × 2.29

= 114.5 years!

- Note that the velocity of spaceship is $v = 0.9 \times 300{,}000$ km/s = 270,000 km/s

Ex.2 A given GPS satellite travels at a velocity (v) of 14,000 km/h. The speed of light (c) is given to be 299,792 km/s.

Calculate the time discrepency between the satellite clock and the GPS clock, over a period of 24 hrs.

Consequently calculate the error in distance measurement between the satellite & GPS device

- we have

$$v = 14{,}000 \text{ km/h} = 14{,}000/(60\times 60) \text{ km/s}$$
$$= 3.8889 \text{ km/s}.$$

One earth day on the satellite is

$$\Delta t_s = \underset{hrs}{24} \times \underset{min}{60} \times \underset{sec}{60} = 86{,}400 \text{ s}$$

$$\therefore \Delta t_e = \frac{\Delta t_s}{\sqrt{1-v^2/c^2}} = \frac{86{,}400}{\sqrt{1-\left(3.8889/299972\right)^2}}$$

$\simeq$ 86400.0000073 seconds

∴ clock on earth move approx 7.3 sec faster.

- Distance measurement error

$$= c \times \Delta t = 299{,}972 \times 7.3\,\mu s$$
$$= 1.98 \text{ km/day}$$

- The distance to satellite is approximately 20,000 km
- The Error is negligible, but the error is cumulative if it is not corrected
- NOTE: There is also another correction as per General Relativity!