U3A Maths

3-Mar-2026 Term 1 / Week 6

Pythagoras Theorem

Review (Home Work)

Mass of Jupiter (M) = 1.9×10^{27} kg

Radius of Jupiter (r) = 71,000 km

$= 71 \times 10^6$ m

(a) Acceleration due to gravity (g) on the surface of Jupiter.

$$F = \cancel{m}g = \frac{G.M.\cancel{m}}{r^2}$$

$$\therefore g = \frac{6.67408 \times 10^{-11} \times 1.9 \times 10^{27}}{(71 \times 10^6)^2}$$

$$= \frac{6.67408 \times 1.9}{71 \times 71} \times \frac{10^{-11} \times 10^{27}}{10^{12}}$$

$$= 0.002515 \times 10^4$$

$$= 25.15 \ m/s^2$$ {Earth (g) = $9.81 \ m/s^2$}

(b) Force exerted by 65 kg mass (weight) on Jupiter

$F (= W) = m \cdot g$

$= 65 \times 25.15$

$\cong 1635$ Newtons (N).

{Note: Weighing scale measures "weight" but displays "mass"}

on Earth $F (= W) = m g$

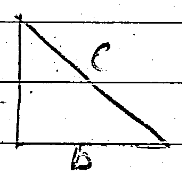
$= 65 \times 9.81$

$= 638$ N

$\therefore$ Approx: $\frac{1635}{638} \simeq 2.56$ times heavier

Pythagoras Theorem

- For a right angled triangle



$$c^2 = a^2 + b^2$$

- Pythagoras (570 - 490 BC)

Even though the above theorem is called Pythagoras Theorem, the theorem itself existed in antiquity. (Rule of 3-4-5)!

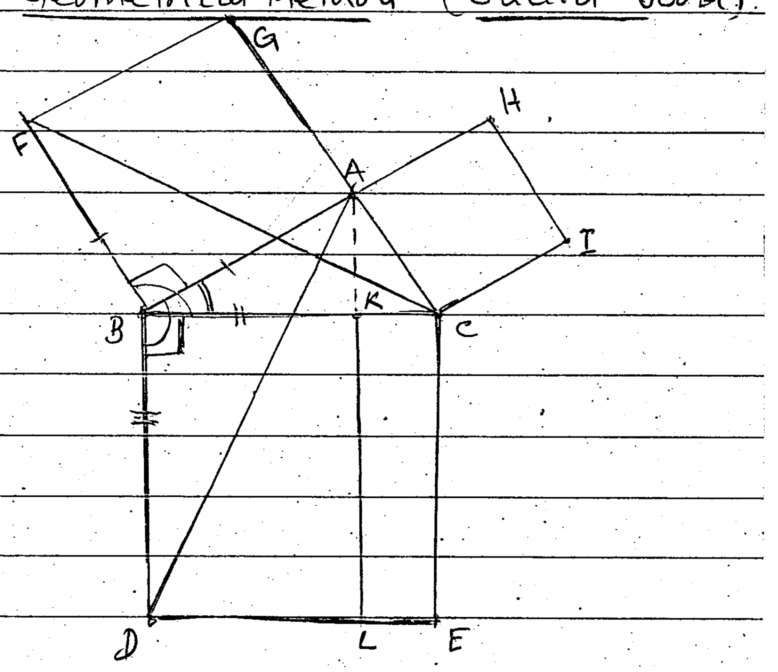
1800 BC - 1600 BC - Egyptians / Babylonians

800 BC - 500 BC - Indian documents

100 BC - Chinese documents

- Proof of Pythagoras theorem

Geometrical Method (Euclid 300 BC)



The objective is to show that

(a) Rect Area BKLD = Square Area ABFG

(b) Rect Area KCEL = Square Area ACIH

Let us prove Part (a)

We have $\triangle ABD \equiv \triangle FBC$

since, $BD = BC$, $AB = FB$ & $\angle ABD = \angle FBC$

We will show that

Area of $(\triangle ABD + \triangle FBC)$

$=$ Area of Rect $BKLD = BD \times BK$

$=$ Area of Square $ABFG = AB \times FB$

We have,

Area of $\triangle ABD = \frac{1}{2} \cdot BD \times BK$.

$\therefore \triangle ABD + \triangle FBC = 2 \times \triangle ABD$

$= \cancel{2} \times \frac{1}{\cancel{2}} BD \times BK$

$= BD \times BK$!

Area of Rect BKLD $\longrightarrow$

Also,

Area of $\triangle FBC = \frac{1}{2} FB \times AB$

$\therefore \triangle FBC + \triangle ABD = 2 \times \triangle FBC$

$= \cancel{2} \times \frac{1}{\cancel{2}} \times FB \times AB$

$= FB \times AB$!

Area of Rect ABFG

Q.E.D.

Part (b)

Home work

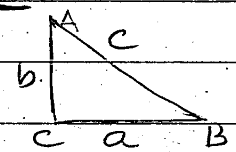
Show that

Area of Rect KCEL = Area of Square AHIC.

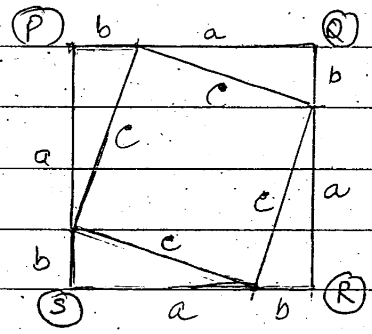
$\therefore$ We have, $BC^2 = AB^2 + AC^2$

Geometric + Algebraic Method

$c^2 = a^2 + b^2$



Construct a square with a side length equal to $(a+b)$ as shown



Area of the Square P-Q-R-S

$= c^2 +$ Area of 4 triangles

$= c^2 + 4 \times \frac{1}{2} a \times b$.

$\therefore$ We have

$(a+b)^2 = c^2 + 4 \times \frac{1}{2} ab$

$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$

$\therefore \; c^2 = a^2 + b^2$!

Trigonometric Method

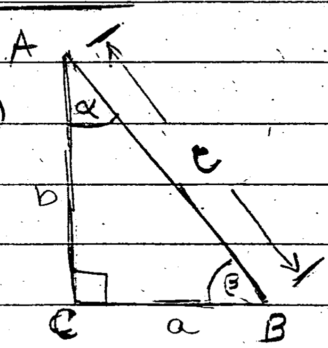
(Einstein's method !)

We have

$\sin(\alpha) = \frac{a}{c} = \cos(\beta)$

Also

$\sin(\beta) = \frac{b}{c} = \cos(\alpha)$



We have

$a = c \sin(\alpha)$ & $b = c \cos(\alpha)$

$\therefore \; a^2 + b^2 = c^2 \sin^2(\alpha) + c^2 \cos^2(\alpha)$

$= c^2 (\sin^2\alpha + \cos^2\alpha) = c^2$

- One of the early use of Pythagoras theorem was the <u>Pythagorean Triple</u>

For. Ex:

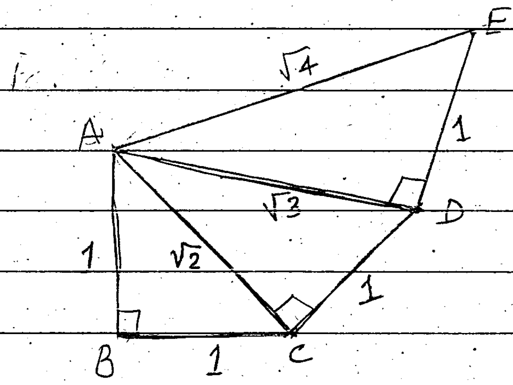
$3-4-5 \Rightarrow 3^2+4^2=5^2$

- Hence, a triangle with sides 3-4-5 is a right angled triangle.

other Triples are (5, 12, 13), (7, 24, 25) etc

- The above can be used to establish a right angle (90°) accurately, with linear measurements.

- Another interesting use of Pythagoras theorem was to calculate square roots!



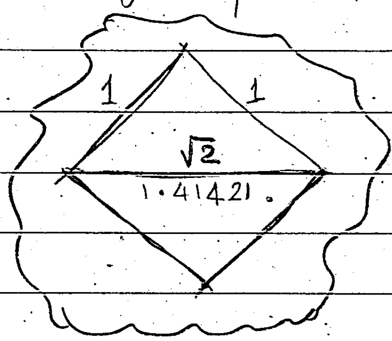
$AC = \sqrt{AB^2+BC^2} = \sqrt{1^2+1^2} = \sqrt{2}$

$AD = \sqrt{AC^2+CD^2} = \sqrt{(\sqrt{2})^2+1^2} = \sqrt{3}$

etc

- The above method was used by Babylonians to find square roots. (1600 BC)

- A Babylonian ~~tablet~~ stone tablet has survived which accurately shows the value of square root of 2



- Of course, Babylonians used a number system with Base 60!

= x =