U3A Maths

17-Mar-2026 Term 1 / Week 8

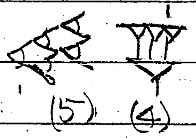
Square Root - Babylonian Method

Review

- Babylonians used Base 60 (Sexagesimal) number system with only 2 symbols

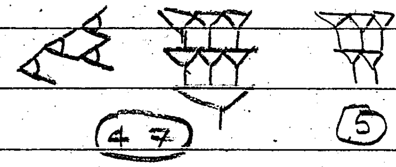
 1 ⇒ Y 10 ⇒ <

 For Ex. 54 is represented by:

 54 ⇒ 

 They had unique symbols for digits 1 to 59. Values greater than 59 was represented by position based system.

For Ex: 47 5 in Base 60.

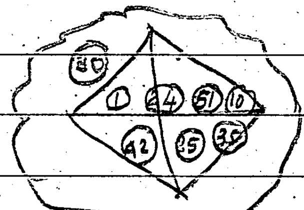


The decimal equivalent is

47 5 $\Rightarrow 60^1 \times 47 + 60^0 \times 5$

$= (2,825)_{10}$

The clay tablet (≅ 1600 BC) with an archiological reference YBC 7289 is as below.



- Last week, we found that Clay tablet numbers

 1 . 24 51 10 (Base 60)

 Corresponds to 1.414212,96...
- Using calculator, $\sqrt{2} = 1.414,21356...$
- The clay tablet value has a precision of 5 decimal digits!

- The clay tablet numbers

 30 42 25 35

 have been interpreted in 2 ways

Interpretation #1

 . 42 25 35

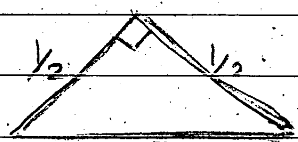
$\Rightarrow 42/60 + 25/60^2 + 35/60^3$

$= 0.70711$

which is $\cong \sqrt{1/2}$ or $1/\sqrt{2}$

In such a case the number 30 is interpreted as:

- 30 $\Rightarrow 30/60 = 1/2$ or 0.5

Also, 

$\sqrt{1/2} = 0.707$

Interpretation #2

30 is interpreted as 30

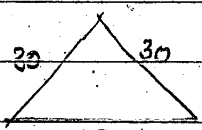
The numbers 42 25 35

as $\overset{60^0}{42} . \overset{60^{-1}}{25}\ \overset{60^{-2}}{35}$

$= 42 \times 60^0 + 25/60 + 35/60^2$

$= 42.4264$

Using Calculator: $30 \times \sqrt{2} = 42.4264$



Square Root – Babylonian Method

Ex.1 Calculate $\sqrt{17}$

Say $x = \sqrt{17}$ where $s = 17$

As a first approximation let $x_0 = 4$

We can refine this value using:

$$x_1 = x_0 + \frac{S - x_0^2}{2x_0}$$

$$= 4 + \frac{17 - 4^2}{2 \times 4}$$

$$= 4 + \frac{1}{8} = \underline{\underline{4.125}}$$

List of Squares

$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
(17) $5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$
etc

Using the calculator

$\sqrt{17} = \underline{4.123\ldots}$

- Quite often, the above calculation gives fairly good (acceptable) results! (upto about 2 digits)
- The process can be continued for better accurcy, however, the calculations become more complex.

Ex.2 Calculate $\sqrt{69}$

Using the list of squares, we have initial estimate $x_0 = 8$

$$x_1 = 8 + \frac{69 - 64}{2 \times 8}$$

$$= 8 + \frac{5}{16} \approx \underline{8.3125}$$

Using Calculator $\sqrt{69} = \underline{8.3066}$

Ex.3

Calculate $\sqrt{47}$

Since 47 is close to 7^2, let us use $x_0 = 7$

$$x_1 = x_0 + \frac{s - x_0^2}{2x_0}$$

$$= 7 + \frac{47 - 7^2}{2 \times 7} = 7 + \frac{(-2)}{2 \times 7}$$

$$= 7 - \frac{2}{2 \times 7} = 7 - \frac{1}{7} = \underline{6.857}$$

Using calculator $\sqrt{47} = \underline{8.8556}$

Ex.4

Calculate $\sqrt{2}$

Using "List of squares", let $x_0 = 1$

$$\therefore x_1 = 1 + \frac{2 - 1^2}{2 \times 1} = \underline{\underline{1.5}}$$

For better precision, we can continue the process:

$$x_2 = x_1 + \frac{S - x_1^2}{2x_1} = 1.5 + \frac{2 - (1.5)^2}{2 \times 1.5}$$

$$= 1.5 + \frac{(-0.25)}{3} = 1.41666\ldots$$

$$x_3 = 1.41666 + \frac{2 - (1.41666)^2}{2 \times 1.41666} = \underline{\underline{1.414213}}\ !!$$

- The method used in Ex. 4, is called "method of Iteration". In fact, even modern computers (calculators) use this method extensively!

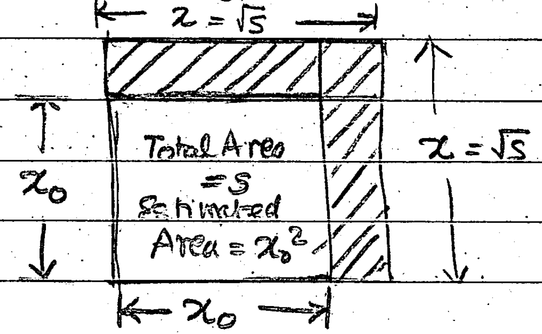
Proof by Geometrical Method

Let $x = \sqrt{S}$

where 'S' is the given value to calculate the square root.

- Using the "List of Squares", let 'x_0' be our inital estimate

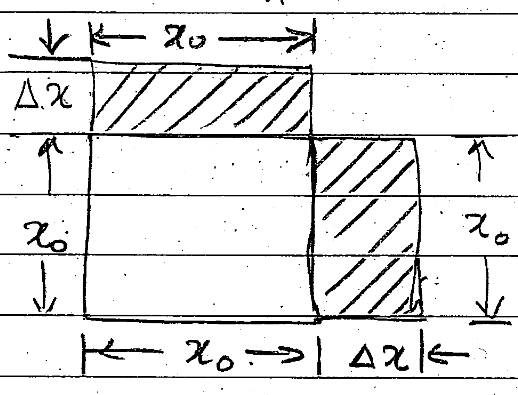
- Graphically, we have.



- The hatched part of the square is, let us say is the "Error"

$\therefore$ Error $= S - x_0^2$

- Let us rewrite the "Error Area" as shown.



- Note that the "Shaded Area" above is equal to the "Error Area".

- Shaded Area $= 2 \times x_0 \times \Delta x$

  Error Area $= (S - x_0^2)$

We have $(S - x_0^2) = 2 . x_0 . \Delta x$

$$\therefore \Delta x = \frac{S - x_0^2}{2x_0}$$

Hence, our new estimate is

$$x_1 = x_0 + \Delta x$$

$$\boxed{x_1 = x_0 + \frac{S - x_0^2}{2x_0}}$$

- The above "algorithm" is same as the "iterative method" proposed by Newton.

- The "Newton's Method" is used extensively to calculate the root of any given algebraic function!