U3A Maths

24-Mar-2026 Term 1/Week 9

Roots of Functions

Review

- Babylonian method (algorithm) for calculating Square Root

$$x = \sqrt{5}$$

Given $x_0 \Rightarrow$ Initial Estimate Using Square Table

$$\boxed{x_1 = x_0 + \frac{5 - x_0^2}{2x_0}}$$

- The above Equation gives surprisingly good results.
- The process can be repeated for better precision

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- We know what is Square Root, but what does the term "Root" mean?
- Let us generalise and extend the term "Root" of a Number to "Root" of a function.
- In fact, we can even extend the term to other roots, such as, cube root, etc and in general n^{th} root.

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- Let us consider a function.

Ex.1. $f(x) = 2x$

Definition of Root

- The "root" of a given function is the value 'x' for which the function value is zero
- In the above Ex,

for $x = 0$

function value $f(0) = 2 \times 0$

$= \underline{0}$

$\therefore$ $x = 0$ is the root of the function $f(x) = 2x$.

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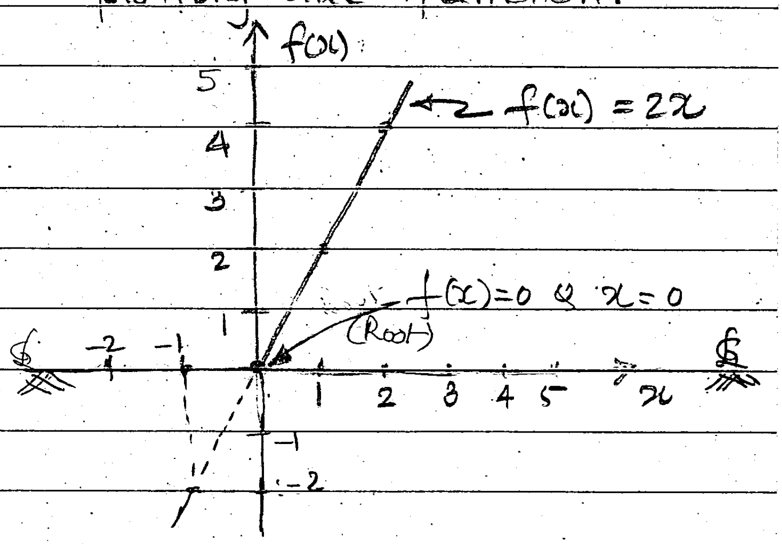
- Let us explore the what this means by plotting the function.



Assuming 'x'-axis as the "Ground Level" (GL), the function values >0 (Positive) are Above Ground and function values <0 (negative) are Below Ground!

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- Hence, $f(x)=0$ represents the starting point of the root(!). Hence, it is called the Root of the function.

Ex.2 Calculate the root of the function
$$f(x) = 2x-4$$

ie, Set $f(x)=0$ and find 'x'

$f(x) \Rightarrow 2x-4=0$

$f(x) \quad \therefore x = 4/2 = \underline{2}$ ← Root

Let us plot the function:

$x \Rightarrow$	0	1	2	3	4	5	...
$f(x) \Rightarrow$	−4	−2	0	2	4	6	...
			↑Root				

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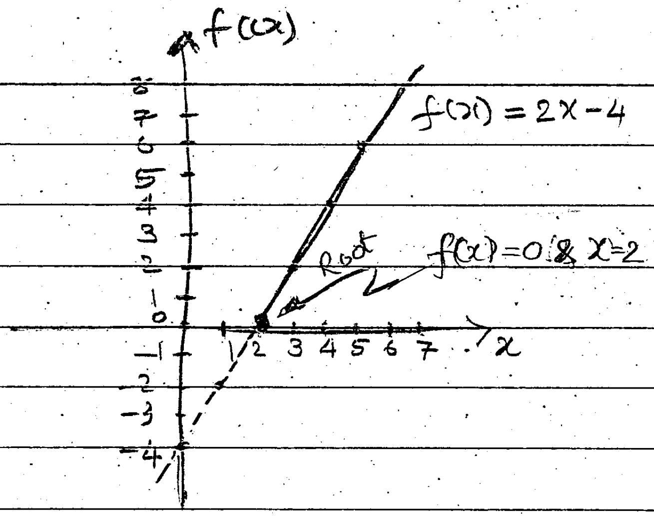

Ex-3 Calculate the root of the function
$$f(x) = x^2-4$$

we have

$$f(x) = x^2-4=0$$
$$\therefore x^2 = 4 \text{ or } x=\sqrt{4}$$
$$= \pm 2$$

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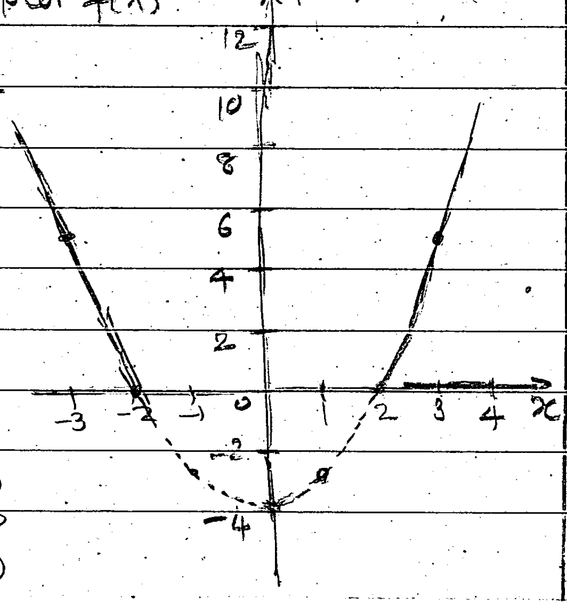
(Cardano 1545)

- Upto about 1500 AD, Europeans considered negative numbers as "Absurd" or "False". Hence, negative results were ignored as impractical numbers! Hence, $\sqrt{4} = +2$!

- Let us plot $f(x)$

x	$f(x)$
0	−4
1	−3
→2	0
3	5
4	12
⋮	⋮
−1	−3
−2	0
−3	5



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- Root of the function (x^2-4), is essentially $\sqrt{4}$

- Similarly, function corresponding to $\sqrt{2}$ is (x^2-2)!

- Hence, the term "Square Root" of a number, is essentially the root of the function (x^2-n)

- Similarly,

$\sqrt[3]{n} \Rightarrow$ function (x^3-n)

$\sqrt[4]{n} \Rightarrow$ function (x^4-n)

<u>Ex. 4</u> Calculate the root of the function
$$f(x) = x^3 - 4$$

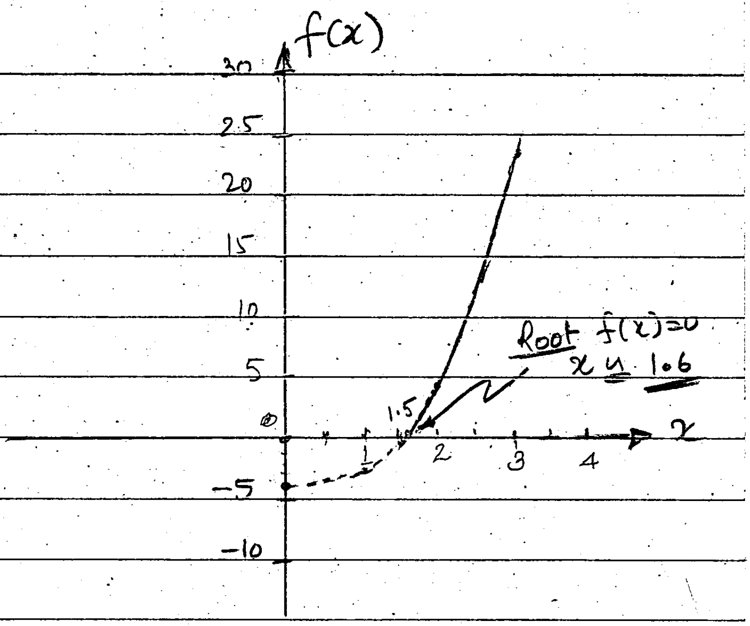
We have $f(x) = x^3 - 4 = 0$

$\therefore x^3 = 4$

or $x = \sqrt[3]{4}$

- We need a calculator to calculate the Root!
- However we can plot the function to find the root!

$x \Rightarrow$	0	1	1.5	2	3	4	...
$f(x) \Rightarrow$	-4	-3	-0.625	4	23	60	...



- The Root $x \simeq 1.6$ (?)
- Very soon we will establish an algorithmic method to calculate cube root, fourth root and any n^{th} root! (quad?)

<u>Home work</u>

- Extend the plot of $f(x) = x^3 - 4$ for negative values of x.
- Check whether there are any more roots.

(<u>Note:</u> In theory, a cube root has 3 roots and at least <u>one</u> is a <u>positive</u> root) (?)

- How do we calculate the other <u>roots</u>?