U3A Maths

21-APR-2026 Term 2 / Week1

Square Root. & Higher Roots

For a given value of 'S' the square root: $x = \sqrt{S}$

Let x_0 be the initial guess (using the "table of squares")

The new estimate of square root

$$x_1 = x_0 + \frac{(S - x_0^2)}{2x_0} \quad \text{— (1)}$$

This algorithm gives surprisingly accurate results within 1 or 2 iterations!

Alternative way to express the above equation is

$$x_1 = \frac{1}{2}\left[x_0 + \frac{S}{2x_0}\right] \quad \text{— (2)}$$

The above equation provides for more efficient computation.

- The above equation(s) can be extended to calculate 3^{rd}, 4^{th} etc and n^{th} root in general.

Cube Root: $x_1 = \left[x_0 + \frac{S - x_0^3}{3x_0^2}\right]$ — (1)(a)

or $x_1 = \frac{1}{3}\left[2x_0 + \frac{S}{x_0^2}\right]$ — (2)(a)

Fourth Root

$$x_1 = \frac{1}{4}\left[3x_0 + \frac{S}{x_0^3}\right]$$

n^{th} Root

$$x_1 = \frac{1}{n}\left[(n-1)x_0 + \frac{S}{x_0^{(n-1)}}\right]$$

Home Work

Ex) Calculate $x = \sqrt[5]{48}$

Perform 2 iterations

let $x_0 = 2$

$1^5 = 1$

$2^5 = 32$

$3^5 = 243$

We have

$$x_1 = \frac{1}{5}\left[4x_0 + \frac{S}{x_0^4}\right]$$

Iter #1

$$x_1 = \frac{1}{5}\left[4 \times 2 + \frac{48}{2^4}\right]$$

$$= \frac{1}{5}\left[8 + \frac{48}{16}\right] = 2.2$$

Iter #2

$$x_2 = \frac{1}{5}\left[4 \times 2.2 + \frac{48}{(2.2)^4}\right]$$

$$= \frac{1}{5}\left[8.8 + \frac{48}{23.4256}\right]$$

$$= 2.1698$$

(Diff: $2.2 - 2.1698 = 0.0302$)

Iter #3

$$x_3 = \frac{1}{5}\left[4 \times 2.17 + \frac{48}{2.17^4}\right]$$

$$= 2.1689$$

(Diff $2.1698 - 2.1689 = 0.0009$)

Check: $(2.1689)^5 = 47.995$

Ex.2 Calculate $x = \sqrt[4]{18}$

We have:

$$x_1 = \frac{1}{4}\left[3x_0 + \frac{S}{x_0^3}\right]$$

$1^4=1$

$2^4=16$

$3^4=81$

Iter#1 let $x_0 = 2$

$$x_1 = \frac{1}{4}\left[3\times 2 + \frac{18}{2^3}\right]$$

$= \underline{2.0625}$ $\quad (2.0625^4 = \underline{18.0957})$

Iter#2

$$x_2 = \frac{1}{4}\left[3\times 2.0625 + \frac{18}{2.0625^3}\right]$$

$= \underline{2.0598}$ $\quad (2.0598^4 = \underline{18.0011})$

Ex.3 $x = \sqrt[3]{128} = \sqrt[3]{64\times 2}$

$= \sqrt[3]{64}\times\sqrt[3]{2}$

$= 4\times\sqrt[3]{2}$

Calculate $\sqrt[3]{2}$

$1^2=1$

$2 \to 2^2=4$

We have:

$$x_1 = \frac{1}{3}\left[2x_0 + \frac{S}{x_0^2}\right]$$

let $x_0 = 1$

Iter#1

$$x_1 = \frac{1}{3}\left[2\times 1 + \frac{2}{1^2}\right]$$

$= 1.3333$ $\quad (1.3333^3 = \underline{2.37})$

Iter#2

$$x_2 = \frac{1}{3}\left[2\times 1.3333 + \frac{2}{1.3333^2}\right]$$

$= 1.2639$ $\quad (1.2639^3 = 2.019)$

Error $= 0.95\%$

$\therefore\ \sqrt[3]{128} = 4\times 1.2639 = \underline{5.0556}$

$(5.0556)^3 = \underline{129.2165}$

Error $= \underline{0.95\%}$ ✓

You can do one more iteration if higher precision is needed.

Ex.4

Calculate $x = \sqrt[4]{50}$

$1^4=1$

let $x_0 = 2.5$ $\qquad 2^4=16$

$\to 51$

$3^4=81$

Iter#1

$$\therefore\ x_1 = \frac{1}{4}\left[3\times 2.5 + \frac{50}{(2.5)^3}\right]$$

$= \underline{2.675}$ $\quad \{2.675^4 = 51.2\}$

Iter#2

$$x_2 = \frac{1}{4}\left[3\times 2.675 + \frac{50}{2.675^3}\right]$$

$= \underline{2.6593}$ $\quad \{2.6593^4 = \underline{50.011}\}$ ✓

—x—

Ex.5 Calculate $x = (5)^{3/4}$

We have $x = (5^3)^{1/4} = (125)^{1/4}$

$\therefore\ x = \sqrt[4]{125}$

Iter#1 let $x_0 = 3.5$ $\qquad 1^4=1$

$$x_1 = \frac{1}{4}\left[3\times 3.5 + \frac{125}{(3.5)^3}\right]$$

$2^4=16$

$3^4=81$

$\to 4^4=256$

$= \underline{3.3539}$

Iter#2

$$x_2 = \frac{1}{4}\left[3\times 3.3539 + \frac{125}{(3.3539)^3}\right]$$

$= \underline{3.3437}$ $\quad$ check $(3.3437)^{4/3} = 4.99996$