U3A Maths

28-APR-2026 Term 2 / Week 5

Roots of Functions

- Roots of numbers (numeric values) are closely associated with "roots of functions"

- For Ex:

$\sqrt{4}$ can be written as the function $x^2 - 4 = 0$

ie, $x^2 = 4$ or $x = \sqrt{4}$

- More formally, we can write

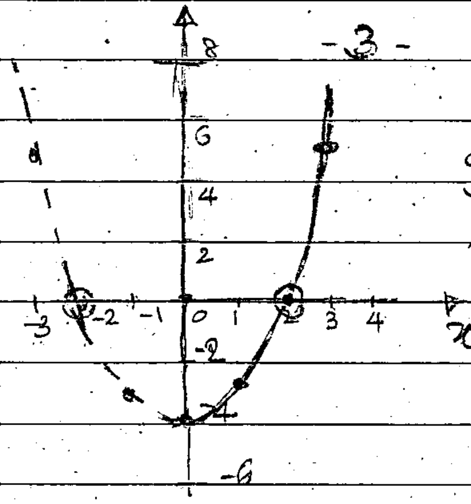
$$f(x) = x^2 - 4$$

- Roots of $f(x)$ can now be defined as the values of 'x' for which the function value is 'zero'.

$f(x) = 0$ or $x^2 - 4 = 0$

- In fact, roots of the function can be found by plotting the function

x :	0	1	2	3	...	-1	-2	-3	...
$f(x)$:	-4	-3	0	5	...	-3	0	5	...



From the graph, we can see that the roots are located at $x = +2$ & $x = -2$

- The above definition of roots can be extended to any given function.

For Ex:

- $f(x) = 2x - 4 = 0$
- $f(x) = x^3 - 27 = 0$
- $f(x) = 2x^2 + 3x + 1 = 0$

- For the present, let us consider the general form of 2^{nd} order equations or quadratic equation and it's practical applications.

- General form

$$f(x) = ax^2 + bx + c$$

The roots of the quadratic can be solved as below:

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{bx}{a} + \frac{c}{a} = 0$$

Keeping the unknowns on L.H.S.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

completing the square on L.H.S

$$x^2 + 2\left(\frac{b}{2a}\right)x + \underbrace{\left(b/2a\right)^2}_{} = \underbrace{\left(\frac{b}{2a}\right)^2}_{} - \frac{c}{a}$$

Add these terms

$$\therefore \left(x + b/2a\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

ie, $\left(x + b/2a\right)^2 = \dfrac{b^2 - 4ac}{4a^2}$

Taking square root, we get

$$\left(x + b/2a\right) = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Simplifying, we get

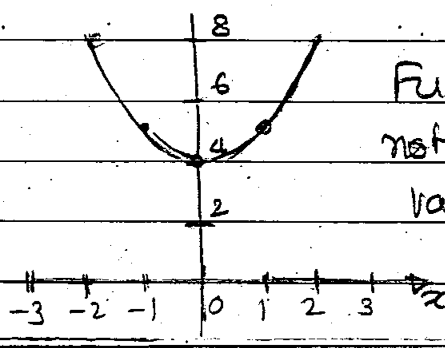
$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

- The algebraic method is more powerful than graphical method!

- For Ex, find the roots of $f(x) = x^2 + 4$

Graphical method

$x =$	-2	-1	0	$+1$	$+2$
$f(x) =$	8	5	4	5	8



Function does not have a zero value!

No Roots!!

Algebraic Method

We have $a = 1$, $b = 0$, $c = 4$

$$\therefore x = \frac{-0 \pm \sqrt{0^2 - 4 \times 1 \times 4}}{2 \times 1}$$

$$= \pm\frac{\sqrt{-16}}{2} = \pm\frac{4\sqrt{-1}}{2}$$

$$= \pm 2 . \sqrt{-1}$$

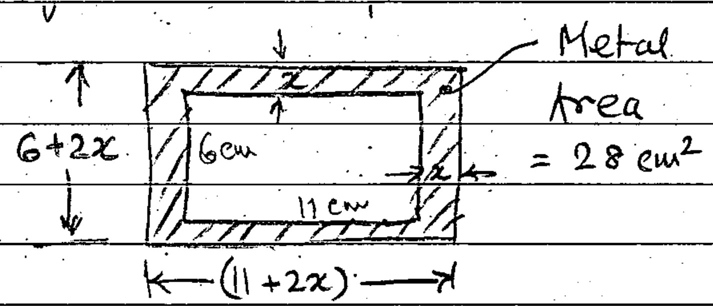
- The roots have a $\sqrt{-1}$ term or "Imaginary" term!

- We will see how to plot such "imaginary roots" in the next few lectures!

Practical Applications of Quadratic Equations

Ex. 1

A frame is to be cut out of a steel plate. To keep the weight down, it is required to have a frame area of 28 cm². Inside of the frame is 6cm × 11 cm. What should be the width 'x' of the metal frame?



-9-

Total Area

$$= (6+2x)(11+2x)$$

$$= 4x^2 + 22x + 12x + 66$$

$$= 4x^2 + 34x + 66$$

Steel Area $= (4x^2 + 34x + 66) - (6 \times 11)$

$$= 4x^2 + 34x.$$

This area must be $= 28\,cm^2$

ie, $4x^2 + 34x = 28$

ie, $f(x) = 4x^2 + 34x - 28 = 0$.

-10-

Solving for 'x' or the root!

$$x = \frac{(-34) \pm \sqrt{(34)^2 - 4(4)(-28)}}{2 \times 4}$$

$$= \frac{(-34) \pm 40.05}{8}$$

∴ Roots are

$x_1 = 0.76525\,cm$ or $-9.25625\,cm$

- Negative value has no relevance in practice,

∴ $x = 0.76525\,cm$

Home work: Using steel area as the function
Steel Area $\Rightarrow f(x) = 4x^2 + 34x$
Plot x & $f(x)$ [y-axis)]. Find 'x' for $f(x) = 28$

-11-

Ex-2 (Home work?)

A 3 hour River cruise goes 15 km upstream and then back again. The river has a current of 2 km/hr. What is the boat speed and how long is the upstream journey?

-12-