U3A Maths

5-May-2026 Term 2/Week 3

Quadratic Functions - Applications

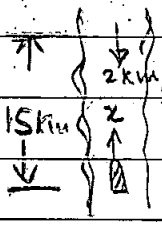
Home Work (Week 2, Ex. 2)

A 3 hour river cruise goes 15 km upstream and then back again. The river has a current of 2 km/hr.

(a) What is the upstream boat speed

(b) How long is the upstream journey

Let the boat speed = x



$$15\left[\frac{(x+2)+(x-2)}{(x-2)(x+2)}\right] = 3$$

$$15(2x) = 3(x-2)(x+2)$$

$$= 3(x^2-2^2)$$

$$= 3(x^2-4)$$

$$\therefore 30x = 3(x^2-4)$$

$$\text{or } 10x = x^2 - 4$$

Finally,

$$x^2 - 10x - 4 = 0$$

We have

$$a=1,\ b=-10,\ c=-4$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Using Physics relationship

Velocity (speed) = $\frac{\text{Distance}}{\text{Time}}$.

Upstream speed = $(x-2)$ km/hr

Downstream speed = $(x+2)$ km/hr

$\therefore$ Upstream Time (hrs) = $\frac{15 \text{ km}}{(x-2) \text{ km/hr}}$

Downstream Time (hrs) = $\frac{15 \text{ km}}{(x+2) \text{ km/hr}}$

$\therefore$ Total time = $\frac{15}{(x-2)} + \frac{15}{(x+2)} = 3$ hrs

$$\therefore 15\left[\frac{1}{(x-2)} + \frac{1}{(x+2)}\right] = 3$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - (4)(1)(-4)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{116}}{2}$$

$$= 10.385 \text{ or } -0.385$$

Neglecting the negative value,

$$x = 10.385 \text{ km/hr}$$

(a) Upstream speed (v_1) = $10.385 - 2$

$$= 8.385 \text{ km/hr}$$

(b) Upstream Journey Duration

$$= \frac{\text{Dist}}{\text{speed}} = \frac{15 \text{ km}}{8.385 \text{ km/hr}}$$

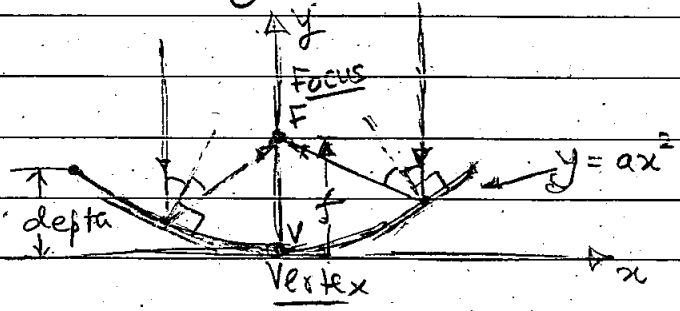
$$= 1.7889 \text{ hrs}$$

[Downstream Time = $\frac{15 \text{ km}}{12.385} = 1.2111$ hrs
Total = 3 hrs!]

Satellite Dish

A satellite dish is a Parabolic dish. In other words, the cross-sectional shape corresponds to a parabola. Hence, it follows the equation

$$y = ax^2$$



- This shape reflects the direct radiation (Light, sound, E.M. waves..)

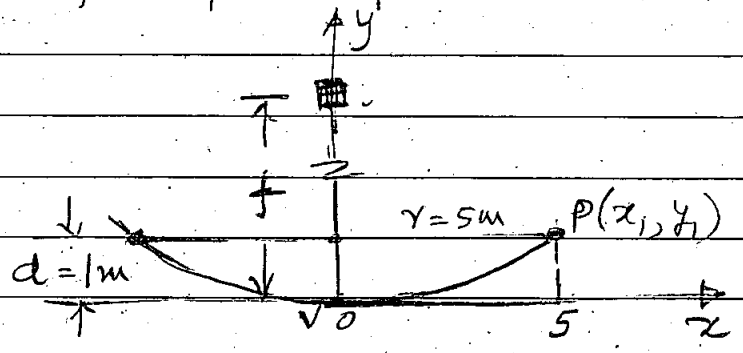
falling on it to the Focal point (F).

- Radio telescope antennas, satellite dishes and sound gathering dishes, use this concept by placing the receiver at the Focal Point

- Distance of the Focal Point to the Vertex (V) is given by

$$f = 1/4a$$

Ex.1 A parabolic dish has a radius of 5m and a depth of 1m. Calculate the distance to focal point from the vertex.



In the above figure the coordinates of the point 'P' are $x_1 = 5$ & $y_1 = 1m$

Equation for parabola is

$$y = ax^2$$

we have, $(1)^2 = a(5)^2$

$$\therefore a = 1/5^2 = (1/25)$$

$\therefore$ Focal point

$$f = 1/4a$$

$$= \frac{1}{4(1/25)} = \frac{25}{4}$$

$$= \underline{6.25\ m}$$

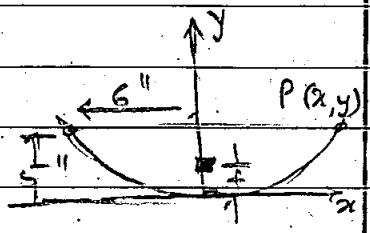
Focal point is 6.25m from the bottom of the dish.

Ex.2 A bird watcher gets a parabola shaped bowl from a cooking store. The diameter of the bowl is 12 inches and the depth is 5 inches. At what distance from the bottom, should the microphone be mounted.

We have

$x_1 = 12/2 = 6''$

$y_1 = 5''$



$\therefore\ y = ax^2 \Rightarrow a = 5/6^2 = (5/36)$

Focal distance from bottom

$= f = \frac{1}{4a} = \frac{1}{4 \times 5/36}$

$= \frac{9}{5} = 1.8$ inches

Homework A hobbiest builds a solar cooker using small mirrors stuck on to a parabolic dish with a diameter of 3m and a depth of 0.5m. Find where the cooker is to be located.

Note: Assuming sun falls on a area of 2 sqm & the cooker area 0.1m x 0.1m the solar

amplification at the cooker will be $\frac{2}{0.1 \times 0.1} = 200$ times!