U3A Maths

12-May-2026 Term 2 / Week 4

Parabola Characteristics

- A parabola is formally defined as a curve, where any point (on the curve) is at an equal distance from,

  - a fixed point (the Focus)

  & - a fixed straight line (the Directrix)

- An example plot of a Parabola is as shown below:

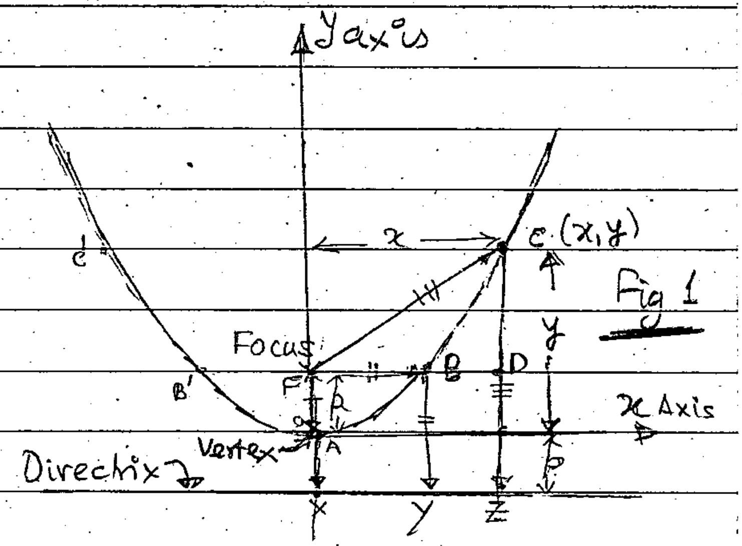

Fig 1

At point A : $FA = AX$
At point B : $FB = BY$
At point C : $FC = CZ$

- Even though "Directrix" is a part of the Parabola definition,

however, it is common to use the horizontal line through 'A' (the Vertex) as the x-axis and the vertical line through 'A' as the 'y' axis.

- Hence, the point 'A' is used as the origin $(0,0)$ for developing the equation for the parabola!

- Hence, the "Directrix" line is invariably ignored when drawing a parabola!!

- However we cannot ignore the 'Directrix' line when

developing the equation for the parabola.

Equation for the Parabola

- As a general case, let us consider the point 'C' on the parabola (Refer to Fig 1)

- Let the coordinates of the general point ('C') be,

  $C(x, y)$

- Let coordinates of the Focus be

  $F(0, p)$

  i.e. Coordinate of point 'X' on the directrix is

  $X(0, -p)$

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Distance $CZ = (y+p)$

Considering ΔFCD in Fig 1,

we have $FD = x$ & $CD = (y-p)$

$\therefore$ Distance $FC = \sqrt{(FB)^2 + (BC)^2}$

$$= \sqrt{x^2 + (y-p)^2}$$

As per the definition of the parabola,

Distance CZ = Distance FC

$$(y+p) = \sqrt{(x^2 + (y-p)^2}$$

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For convenience, let square both sides;

$$\therefore \quad (y+p)^2 = x^2 + (y-p)^2$$

ie, $\cancel{y^2} + 2yp + p^2 = x^2 + \cancel{y^2} - 2yp + p^2$

Re-arranging,

$$4yp = x^2$$

or $\boxed{y = \left(\frac{1}{4p}\right) \cdot x^2}$

where "p" is distance from the "Focus" to the "Vertex" of the Parabola.

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In practice, it is common to write the equation of Parabola as,

$$\boxed{y = ax^2}$$

where 'a' is a constant!

Of course, we now know that

$$a = \frac{1}{4p} \quad \text{or} \quad p = \frac{1}{4a}$$

where 'p' is distance between the "Focus" and the "Vertex"

- A general equation for the parabola is normally written as $ax^2 + bx + c$

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- The values of 'b' & 'c' only shift the position of the 'vertex' relative to the origin; but the shape & characteristics of the parabola remain the same!
- The distance of the "Focus" to the "Vertex" is still given by

$$p = \frac{1}{4a}$$

- A parabola has an amazing property:
  "Any ray parallel to axis of symmetry falling on the surface, gets reflected to the Focus"

- Conversely, rays from a source located at the focus will reflect off the surface as parallel rays!

- Hence, Parabolic shape is used for:
  - Satellite & Radar dishes
  - Concentrating solar rays (to Focus)
  - Reflector on spot lights & Torches

Ex 1 Given the equation $y = 0.25x^2$, plot the parabola for $x = 0, 1, 2, 3, 4$ & 5. Also, prove that a ray of light falling on the surface at $x = 3$, will reflect on to the Focus.

$x \Rightarrow$	0	1	2	3	4	5
$y \Rightarrow$	0	0.25	1	2.25	4	6.25

The focus distance is

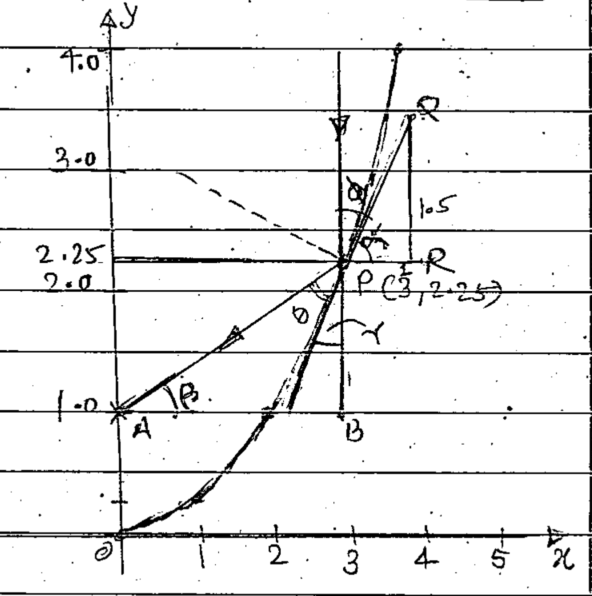
$$p = \frac{1}{4a} = \frac{1}{4 \times 0.25} = 1$$

i.e. distance of '1 unit' from the vertex.

Also, the equation for the "slope" of the tangent is;

$$\text{Slope} = \frac{dy}{dx} = \frac{d}{dx}(ax^2) = 2ax$$

Equation for slope $\Rightarrow 2 \times 0.25 \times x = 0.5x$



We need to show that $\angle\theta = \angle\phi$

Slope of Tangent at $P = 0.5x$

$= 0.5 \times 3 = 1.5$

$\therefore \tan(\alpha) = 1.5$ or $\alpha = 56.31°$

Also $\tan(\beta) = \frac{PB}{AB} = \frac{(2.25 - 1)}{3} = \frac{1.25}{3}$

$= 0.416667$

$\therefore \beta = \tan^{-1}(0.416667) = 22.62°$

We have, $\phi = 90° - \alpha = 90 - 56.31° = \underline{33.69°}$

Also, Angle $\gamma = \phi = 33.69°$

From $\triangle ABP \Rightarrow \beta + \theta + \gamma = 90°$

$\therefore \theta = 90° - \beta - \gamma = 90° - 22.62° - 33.69°$

we have $\underline{\theta = \phi}$ $= \underline{33.69°}$

Home Work

Show that incident & reflected angles are equal at $x = 4$.