U3A Maths

23-Jun-2026 Term 2 / Week10

Function Plotting

Review of Complex Numbers

- Comprehensive (Complex!) numbers include the square root of negative numbers, to make the set of numbers "closed" for all arithmetic operations.

- All square roots of negative numbers can be represented by including, just $\sqrt{-1}$ in the set!

For Ex: $\sqrt{-4} = \sqrt{(-1)(4)} = \sqrt{(-1)} \cdot \sqrt{4}$
$= (\sqrt{-1})\,2$

Using the symbol $i = \sqrt{-1}$, provides for convenient representation, for Ex,
$\sqrt{-4} = i2$, $\sqrt{-9} = i3$, $\sqrt{-121} = i11$ etc

- Hence, the general form of complex numbers can be written as:

$$\underbrace{\overline{z}}_{\text{Complex}} = (\underbrace{x}_{\text{Real Part}} + \underbrace{iy}_{\text{Imaginary Part (!)}})$$

- "Imaginary" part is essentially a square root of a negative number.

Ex: $\overline{z} = (x + iy) = (4 + i3)$
$= (4 + \sqrt{-9})$

- Since, we don't know what is $\sqrt{-9}$, we leave it as it is. However,

$$(\sqrt{-9})(\sqrt{-9}) = (-9)^{1/2}(-9)^{1/2}$$
$$= -9^1 = -9$$

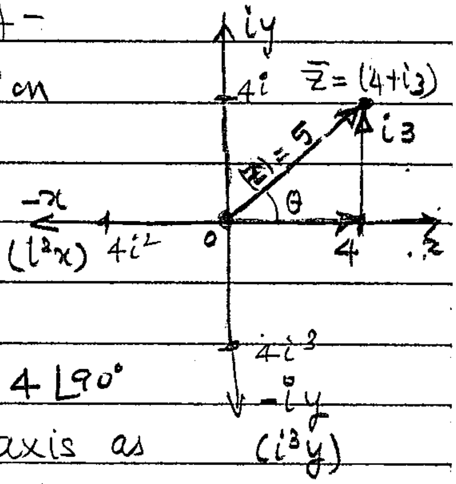
In other words,

$$iy \times iy = i^2 y$$
$$= (\sqrt{-1})^2 y = -y$$

$\therefore$ we have, $i^2 = -1$, $i^3 = -i$ & $i^4 = 1$ etc

- Graphically, we can represent a complex number on 'x' and 'iy' axis

- Multiplication by 'i' rotates the value by $90°$!



For Ex: $4i$ is $4\angle 90°$ using '$+x$' axis as the reference axis.

- Complex Number representation

Cartesean form
$\overline{z} = (x + iy)$ $\Rightarrow$ Ex $\overline{z} = (4 + i3)$

Polar Form
$\overline{z} = |z| \angle\theta$ $\Rightarrow$ Ex: $\overline{z} = (4 + i3)$
where $|z| = \sqrt{x^2 + y^2}$ $= 5\angle 36.86°$
$\theta = \tan^{-1}(y/x)$

Euler's form

$$\overline{z} = |z|\, e^{i\theta}$$

• Euler's form is mathematically versatile, due to Euler's formula, namely,

$$\boxed{e^{i\theta} = \cos(\theta) + i\sin(\theta)}$$

$\therefore \bar{Z} = (x + iy)$

$= |Z|\, e^{i\theta}$, where $|Z| = \sqrt{x^2+y^2}$, $\theta = \tan^{-1}(y/x)$

$= \underbrace{|Z|\cos(\theta)}_{x} + i\, \underbrace{|Z|\sin(\theta)}_{y}$

For Ex: $\bar{Z} = (4 + i3)$

$= 5\, e^{i36^\circ 86}$

$= 5\,[\cos(36^\circ 86) + i\sin(36^\circ 86)]$

$= 5\,[0.8 + i\,0.6]$

$= \underline{4 + i3}$

• Finally, Euler form can be used to prove that

$(-4) \times (-4) = +16$!

$-4 \Rightarrow 4e^{i180^\circ}$

$\therefore (-4)\times(-4) = 4e^{i180} \times 4e^{i180}$

$= 16e^{i360^\circ}$ or $4e^{i0^\circ}$

$= 16\,[\cos(0^\circ) + i\sin(0^\circ)]$

$= 16\,[1 + \cancel{i\times 0}]$

$= \underline{16}$ or $\underline{+16}$!

• Similarly, we can show that

$(-4)\times(+4) = \underline{-4}$

[Home work!]

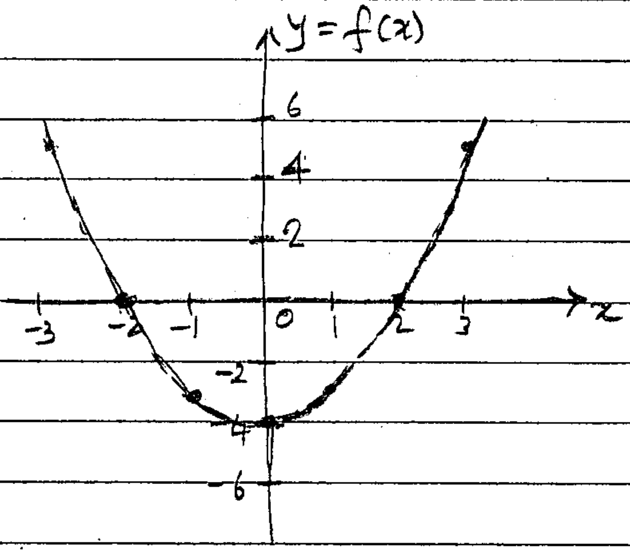
Function Plots

• Let us first familiarise with 'traditional' or 'Real' function plots

Ex.1 Plot $f(x) = x^2 - 4$

x	$f(x)$
-3	$(-3)^2 - 4 = 9 - 4 = 5$
-2	$(-2)^2 - 4 = 4 - 4 = 0$
-1	$(-1)^2 - 4 = 1 - 4 = -3$
0	$(0)^2 - 4 = -4$
1	$(1)^2 - 4 = 1 - 4 = -3$
2	$(2)^2 - 4 = 4 - 4 = 0$
3	$(3)^2 - 4 = 9 - 4 = 5$

etc



• The above function is simple to plot.
Also, $f(x) = 0$ at $x = +2$ & -2 which are called the Roots of $f(x)$.

• Homework

Plot $f(x) = x^2 + 4$ and find the roots!

- Let us now consider function of two variables

For Ex. $f(x,y) = x^2 + y^2 + 4$

Ex.2 Plot the function

for $x = -4$ to $+4$

$y = -4$ to $+4$

Use step size = 1

Hence, we have 'x' value as

$x = -4, -3, -2, -1, 0, 1, 2, 3, 4$

$y = -4, -3, -2, -1, 0, 1, 2, 3, 4$

∴ We have 9 value for 'x' and 9 values for 'y',

i.e. a total of $9 \times 9 = 81$

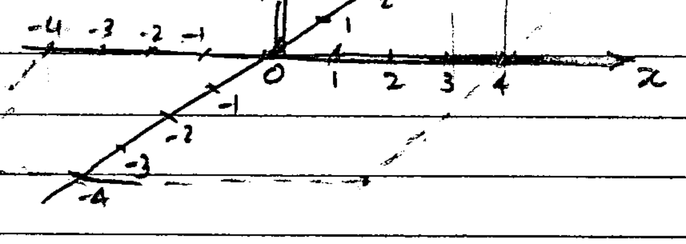
Let us calculate the function values

x	y	$f(x,y)$
−4	0	$(-4)^2+0^2+4 = 20$
−3	0	$(-3)^2+0^2+4 = 13$
−2	0	$(-2)^2+0^2+4 = 8$
−1	0	$(-1)^2+0^2+4 = 5$
0	0	$(0^2)+(0)^2+4 = 4$
+1	0	$(1)^2+(0)^2+4 = 5$
+2	0	$(2)^2+(0)^2+4 = 8$
+3	0	$(3)^2+(0)^2+4 = 13$
+4	0	$(4)^2+(0)^2+4 = 20$
−4	1	$(-4)^2+(1)^2+4 = 21$
−3	1	$(-3)^2+(1)^2+4 = 14$
−2	1	$(-2)^2+(1)^2+4 = 9$
−1	1	$(-1)^2+(1)^2+4 = 6$
0	1	$(0)^2+(1)^2+4 = 5$

etc.

x & y require 2 axes,

∴ $f(x,y)$ has to be the 3rd axis!



- The above plot shows function values for

$x = -4$ to 4 for $y = 0$

- We need to repeat the process for $y = 1, 2, 3, 4$ & $-1, -2, -3$ & -4

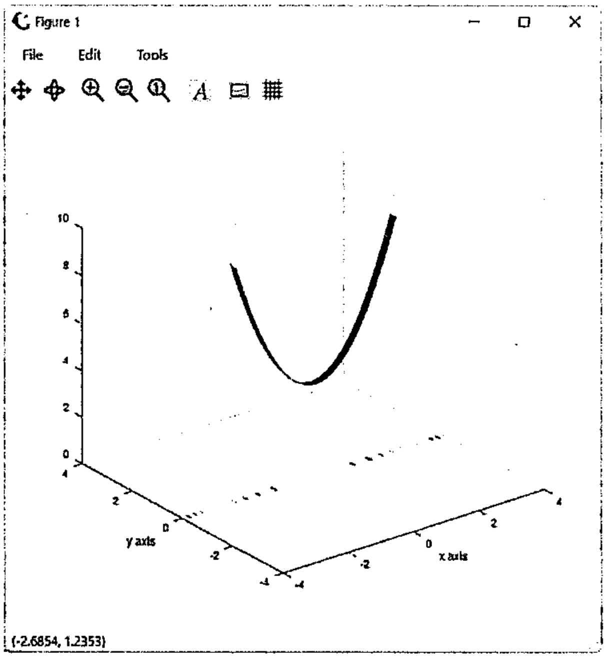
- The final plot is actually a 3-dimensional surface!

- It is difficult plot such functions manually. Hence, specialised plotting software is required to visualise the the functions with 2 variables.

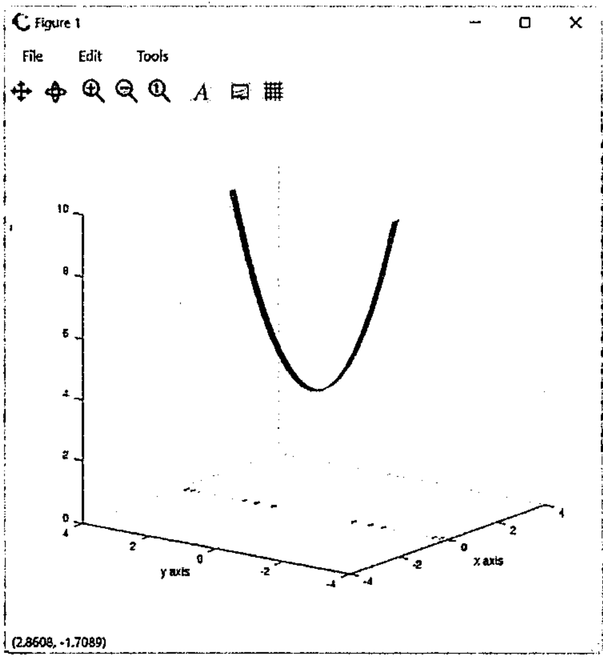
- See attached drawings, which have been produced using 'Gnu Octave' software.

- How do we plot, say $f(x,y,z) = x^2 + y^2 + z^2 + 8$?

Plot of f(x,y) = x^2 +y^2+4 (y = -0.1 to +0.1)



Plot of f(x,y) = x^2 +y^2+4 (x = -0.1 to +0.1)



Plot of f(x,y) = x ^2 +y^2+4 (x = -4 to +4 step 0.25 y = -4 to +4 step 0.25)