U3A Maths

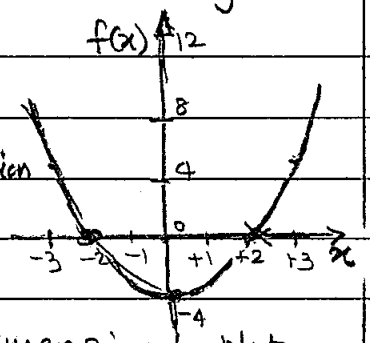
30-Jun-2026 Term 2 / Week 11

Complex Function Plotting

Review - Real Function Plotting.

Ex.1 $f(x) = x^2 - 4$

- Single variable function
- $f(x)$ is a curve, which can be plotted as a 2-dimensional plot



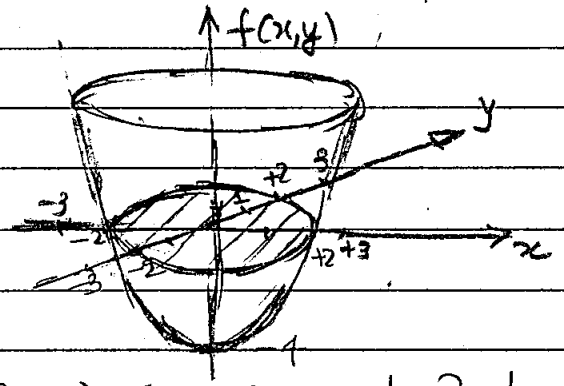
- $f(x) = 0$, correspond to the roots of the function.

The roots are $x^2 - 4 = 0$

$\therefore x = \sqrt{4} = -4$ or 4

Ex.2 : $f(x, y) = x^2 + y^2 - 4$

- Function with 2 variables x & y
- $f(x,y)$ is a curved surface, which can be plotted as a 3-dimensional plot



- $f(x,y) = 0$, correspond to the roots of the function.

The roots are $x^2 + y^2 - 4 = 0$

or $x^2 + y^2 = 4$

- The roots correspond to a circle, shown as "hatched" area in the figure.
- Two-variable functions can be used to generate a variety of interesting and mind boggling surfaces!
- Functions with 3 or more variables cannot be plotted in 3-dimensions! This illustrates the limitations of geometry!
- However, algebra has no such limitations!

For Ex: $f(x, y, z) = x^2 + y^2 + z^2 - 4$

Complex Function Plotting

- Let us consider the complex function,

$f(\bar{z}) = \bar{z}^2 - 4$

where $\bar{z}$ is a complex variable

ie, $\bar{z} = (x + iy)$

Real Part, Imaginary Part

or $\bar{z} = |z| e^{i\theta}$

Magnitude, Angle

- Since, the variable $\bar{z}$ is complex, $f(\bar{z})$ is also complex!

Let us say

$f(\bar{z}) = \bar{w} = (u + iv)$

Real Part, Im. Part

ie, $\bar{w} = (u+iv) = |w| e^{i\phi}$

Magnitude, Angle

where $|w| = \sqrt{u^2+v^2}$

& $\phi = \tan^{-1}(v/u)$

- Note

Variable is $\bar{z} = (x+iy)$

Function value is $\bar{w} = (u+iv)$

$f(\bar{z})$

- To plot complex functions we need 4 dimensions!
- We can plot using three dimensions, by considering only one value of the

function output, namely,

u, v, $|w|$ or ϕ

Real Part, Imh. Part, Mag., Angle.

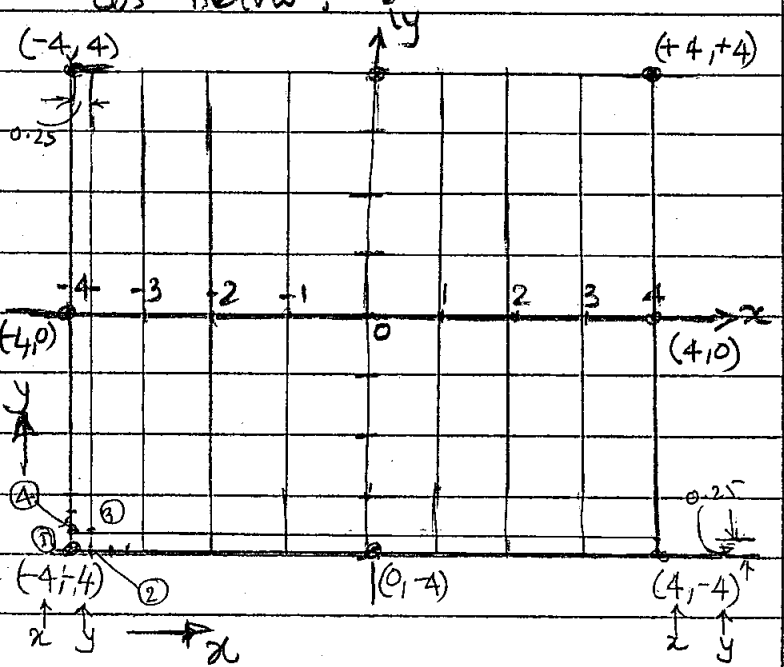
- The most common choice being the "Magnitude" and the "Real Part".
- Let us first calculate some function values for our function

$f(\bar{z}) = \bar{z}^2 - 4$

- As in the case of 2-variable function, we need to consider a "matrix" or a two dimensional

array of value (ie, for x & y).

- Let us organise the values as below:



- The obtain a smoother plot, let us assume the step size = 0.25
- Total No. of points = 33 x 33 = 1089 !!

- For convenience, let us assume (x=-4, y=-4) as our starting point, and calculate function values at points ①, ②, ③, & ④ as shown in the figure.

At point ①: $x = -4$, $y = -4$

$\therefore \bar{z} = (x+iy) = (-4-i4)$

$f(\bar{z}) = \bar{z}^2 - 4$

$= (-4-i4)^2 - (4+i0)$

$= (-4)^2 + 2(-4)(-i4) + (-i4)^2 - (4+i0)$

$= 16 + i32 - 16 - 4 - i0$

$= (-4 + i32)$

At point ② : $x = -3.75$, $y = -4$

$\bar{z} = (-3.75 - i4)$

$f(\bar{z}) = \bar{z}^2 - 4 = (-3.75 - i4)^2 - 4$

$= (-3.75)^2 + 2(-3.75)(-i4) - (i4)^2 - 4$

$= 14.0625 + i30 - 16 - 4$

$= (-5.9375 + i30)$

**Home work **

Similarly, we can calculate

Point ③ $\Rightarrow$ $\bar{z} = (-3.75 - i3.75)$

$f(\bar{z}) = (-4 + i28.1250)$

Point ④ $\Rightarrow$ $\bar{z} = (-4 - i3.75)$

$f(\bar{z}) = (-2.0625 + i30)$

We can verify the above values from computer output

- Plot of $f(z) = \bar{z}^2 - 4$ is as shown on the next page. (Bottom Figure)
- Note that the roots $(f(\bar{z}) = 0)$ are:

$\bar{z}_1 \Rightarrow x = -2, \; y = 0$

$\bar{z}_2 \Rightarrow x = +2, \; y = 0$

- Similarly, we can plot the function $f(z) = \bar{z}^2 + 4$ (Top Figure)
- This function has imaginary roots!
- Note that the roots $(f(\bar{z}) = 0)$ are:

$\bar{z}_1 \Rightarrow x = 0, \; y = -2$

$\bar{z}_2 \Rightarrow x = 0, \; y = +2$

Imaginary Roots !!

Columns 1 through 5:

-4.0000 + 32.0000i (1)	-5.9375 + 30.0000i (2)	-7.7500 + 28.0000i	-9.4375 + 26.0000i	-11.0000 + 24.0000i
-2.0625 + 30.0000i (4)	-4.0000 + 28.1250i (3)	-5.8125 + 26.2500i	-7.5000 + 24.3750i	-9.0625 + 22.5000i
-0.2500 + 28.0000i	-2.1875 + 26.2500i	-4.0000 + 24.5000i	-5.6875 + 22.7500i	-7.2500 + 21.0000i
1.4375 + 26.0000i	-0.5000 + 24.3750i	-2.3125 + 22.7500i	-4.0000 + 21.1250i	-5.5625 + 19.5000i
3.0000 + 24.0000i	1.0625 + 22.5000i	-0.7500 + 21.0000i	-2.4375 + 19.5000i	-4.0000 + 18.0000i
4.4375 + 22.0000i	2.5000 + 20.6250i	0.6875 + 19.2500i	-1.0000 + 17.8750i	-2.5625 + 16.5000i
5.7500 + 20.0000i	3.8125 + 18.7500i	2.0000 + 17.5000i	0.3125 + 16.2500i	-1.2500 + 15.0000i
6.9375 + 18.0000i	5.0000 + 16.8750i	3.1875 + 15.7500i	1.5000 + 14.6250i	-0.0625 + 13.5000i
8.0000 + 16.0000i	6.0625 + 15.0000i	4.2500 + 14.0000i	2.5625 + 13.0000i	1.0000 + 12.0000i
8.9375 + 14.0000i	7.0000 + 13.1250i	5.1875 + 12.2500i	3.5000 + 11.3750i	1.9375 + 10.5000i
9.7500 + 12.0000i	7.8125 + 11.2500i	6.0000 + 10.5000i	4.3125 + 9.7500i	2.7500 + 9.0000i
10.4375 + 10.0000i	8.5000 + 9.3750i	6.6875 + 8.7500i	5.0000 + 8.1250i	3.4375 + 7.5000i
11.0000 + 8.0000i	9.0625 + 7.5000i	7.2500 + 7.0000i	5.5625 + 6.5000i	4.0000 + 6.0000i
11.4375 + 6.0000i	9.5000 + 5.6250i	7.6875 + 5.2500i	6.0000 + 4.8750i	4.4375 + 4.5000i
11.7500 + 4.0000i	9.8125 + 3.7500i	8.0000 + 3.5000i	6.3125 + 3.2500i	4.7500 + 3.0000i
11.9375 + 2.0000i	10.0000 + 1.8750i	8.1875 + 1.7500i	6.5000 + 1.6250i	4.9375 + 1.5000i
12.0000 + 0i	10.0625 + 0i	8.2500 + 0i	6.5625 + 0i	5.0000 + 0i
11.9375 - 2.0000i	10.0000 - 1.8750i	8.1875 - 1.7500i	6.5000 - 1.6250i	4.9375 - 1.5000i
11.7500 - 4.0000i	9.8125 - 3.7500i	8.0000 - 3.5000i	6.3125 - 3.2500i	4.7500 - 3.0000i
11.4375 - 6.0000i	9.5000 - 5.6250i	7.6875 - 5.2500i	6.0000 - 4.8750i	4.4375 - 4.5000i
11.0000 - 8.0000i	9.0625 - 7.5000i	7.2500 - 7.0000i	5.5625 - 6.5000i	4.0000 - 6.0000i
10.4375 - 10.0000i	8.5000 - 9.3750i	6.6875 - 8.7500i	5.0000 - 8.1250i	3.4375 - 7.5000i
9.7500 - 12.0000i	7.8125 - 11.2500i	6.0000 - 10.5000i	4.3125 - 9.7500i	2.7500 - 9.0000i
8.9375 - 14.0000i	7.0000 - 13.1250i	5.1875 - 12.2500i	3.5000 - 11.3750i	1.9375 - 10.5000i
8.0000 - 16.0000i	6.0625 - 15.0000i	4.2500 - 14.0000i	2.5625 - 13.0000i	1.0000 - 12.0000i
6.9375 - 18.0000i	5.0000 - 16.8750i	3.1875 - 15.7500i	1.5000 - 14.6250i	-0.0625 - 13.5000i
5.7500 - 20.0000i	3.8125 - 18.7500i	2.0000 - 17.5000i	0.3125 - 16.2500i	-1.2500 - 15.0000i
4.4375 - 22.0000i	2.5000 - 20.6250i	0.6875 - 19.2500i	-1.0000 - 17.8750i	-2.5625 - 16.5000i
3.0000 - 24.0000i	1.0625 - 22.5000i	-0.7500 - 21.0000i	-2.4375 - 19.5000i	-4.0000 - 18.0000i
1.4375 - 26.0000i	-0.5000 - 24.3750i	-2.3125 - 22.7500i	-4.0000 - 21.1250i	-5.5625 - 19.5000i
-0.2500 - 28.0000i	-2.1875 - 26.2500i	-4.0000 - 24.5000i	-5.6875 - 22.7500i	-7.2500 - 21.0000i
-2.0625 - 30.0000i	-4.0000 - 28.1250i	-5.8125 - 26.2500i	-7.5000 - 24.3750i	-9.0625 - 22.5000i
-4.0000 - 32.0000i	-5.9375 - 30.0000i	-7.7500 - 28.0000i	-9.4375 - 26.0000i	-11.0000 - 24.0000i

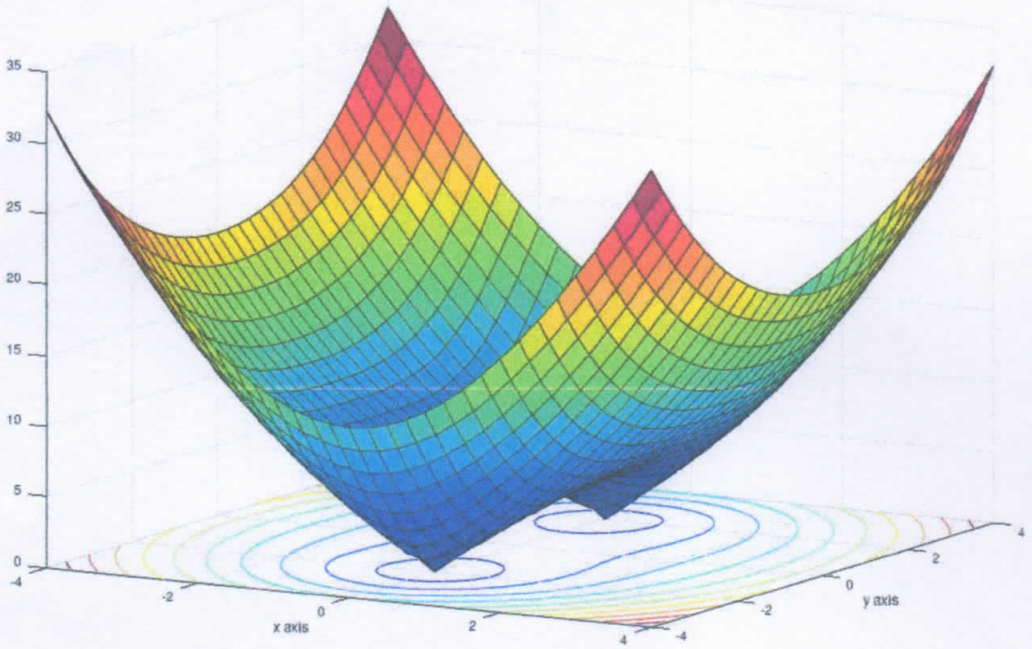
Columns 6 through 10:

-12.4375 + 22.0000i	-13.7500 + 20.0000i	-14.9375 + 18.0000i	-16.0000 + 16.0000i	-16.9375 + 14.0000i
-10.5000 + 20.6250i	-11.8125 + 18.7500i	-13.0000 + 16.8750i	-14.0625 + 15.0000i	-15.0000 + 13.1250i

Figure 1

File Edit Tools

Plot of z^2+4 (Roots at x=0, y=-2 and x=0 and y=+2)



Plot of z^2-4 (Roots at x=-2, y=0 and x=+2, y=0)