U3A Maths

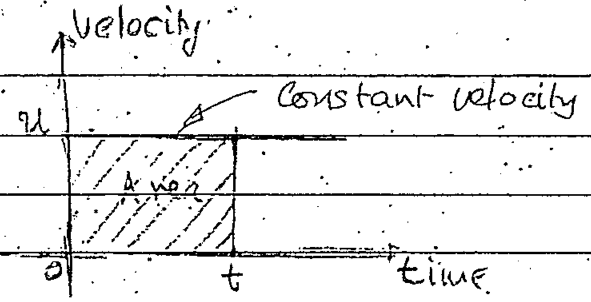
26-May-2026 Term2 / Week6

Projectile Motion

- Let us first derive Equations of Motion with a view to solve Projectile Motion.

- Let us first consider an object moving at a constant velocity (say 'u' meters/sec)

- The distance (s) travelled by the object for a given time (t) can be calculated as below.

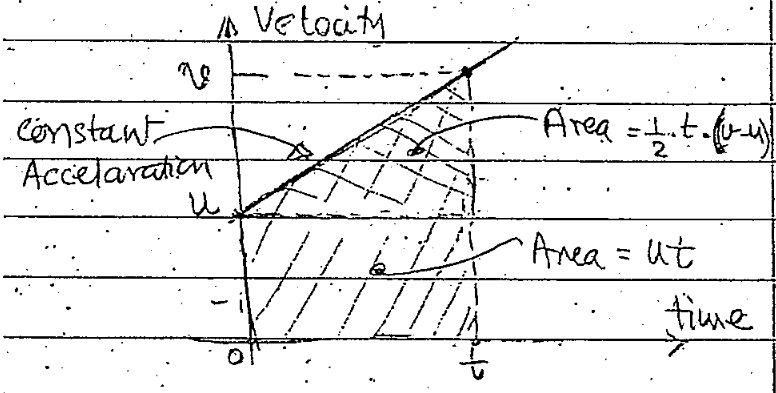


Area = Distance travelled (s)

$\therefore \boxed{S = u \times t}$ — Eqn (1)

- Let us now say that the object is moving at constant velocity (u) at t=0 sec. At this time a force acts on the object and increases the velocity to a new value (v) in a given time 't', as shown in the Figure below.

- Let us now calculate the distance travelled.



Again, Area = Distance travelled (s)

$\therefore S = ut + \frac{1}{2}(v-u)t$

Let us define;

Accelaration (a) = $\frac{\text{Change of velocity}}{\text{time}}$

i.e., $a = \frac{(v-u)}{t}$ — Eqn (2)

$\therefore (v-u) = at$

Substituting, we get

$S = ut + \frac{1}{2}(at)(t)$

$\boxed{S = ut + \frac{1}{2}at^2}$ — Eqn (3)

Note: if $u=0$ then $S = \frac{1}{2}at^2$

- Another useful equation is obtained by substituting for 't' in Eqn (3) from Eqn (2).

We have, $a = \frac{v-u}{t}$

$\therefore t = \frac{(v-u)}{a}$

Substituting in Eqn (2)

$$S = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\cdot\left(\frac{v-u}{a}\right)^2$$

$$= \frac{u(v-u)}{a} + \frac{1}{2a}(v-u)^2$$

Multiplying both sides by "2a"

$$2as = 2u(v-u) + (v-u)^2$$

$$= 2\cancel{uv} - 2u^2 + v^2 - 2\cancel{vu} + u^2$$

$$\therefore \boxed{2as = v^2 - u^2} \text{ - Eqn (4)}$$

• The above equations can be used to solve so many problems involving motion of objects, including Projectiles

Ex. 1. A horse accelarates steadily from rest at $4\,m/s^2$ for 3 seconds. What is its final velocity and the distance travelled.

1. Given, $u = 0$, $a = 4$ & $t = 3$

We have, (Eqn (2)) $a = \frac{v-u}{t} = \frac{v-0}{t}$

$\therefore v = at = 4 \times 3 = 12\,m/s$

$$S = ut + \tfrac{1}{2}at^2$$

$$= 0 + \tfrac{1}{2} \times 4 \times 3^2$$

$$= 18\,m$$

Ex. 2 A car travelling at 30 m/s accelarates steadily at $5\,m/s^2$ for a distance of 70m. What is the final velocity of the car?

We have, (Eqn (4))

$$2as = v^2 - u^2$$

Given $u = 30$, $a = 5$ & $S = 70$

$\therefore 2 \times 5 \times 70 = v^2 - 30^2$

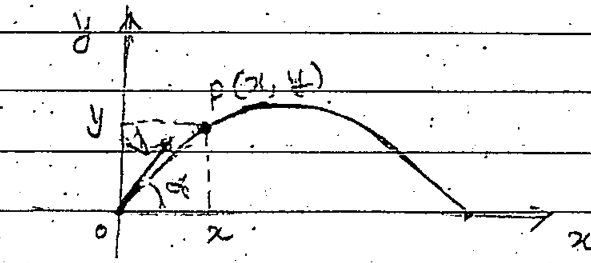
$\therefore v^2 = 700 + 900 = 1600$

$\therefore v = \sqrt{1600} = 40\,m/s$

Note: $\left(\frac{40 \times 3600}{1000}\,km/h\right)$ (144 km/h !)

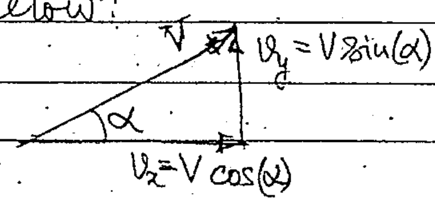
Projectile Model

The following graphical model is useful to solve problems involving projectiles.



• Note that the 'time' is not represented in the above graph!

• The velocity of the object can be resolved along "x" & "y" axes as below:



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- The position of the object at any (given) time is given the coordinates 'x' & 'y' (at say point 'P')

- We need to apply "Equations of Motion" for directions 'x' and 'y' separately.

- For Horizontal Motion

'x' Distance: $x = v_x \cdot t = V\cos(\alpha)\, t$

- For Vertical Motion

'y' Distance: $y = v_y t + \frac{1}{2} a t^2$

$= V\sin(\alpha)\, t + \frac{1}{2} a t^2$

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- The acceleration is the "Acceleration due to Gravity" (g) ($a = -9.81\ m/s^2$) or ($a = -g$)

- The value of 'a' is negative since it is opposite to the assumed positive direction for 'y'.

(Note:
The equation is often written as

$y = V\sin(\alpha)\, t - \frac{1}{2} g t^2$

so that a positive value for 'g' can be used.)

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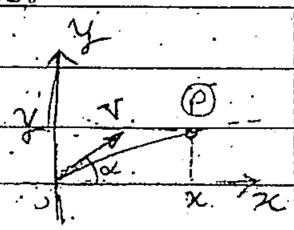
Ex. 3

A golfer hits a ball with a velocity of 45 m/s at an angle of 20° to the horizontal. Find the position of the ball after 2 secs.

Given: $V = 45\ m/s$

$\alpha = 20°$

$t = 2\ s$



Also, $a = -9.81\ m/s^2$

we have

$x = V\cdot\cos(\alpha)\cdot t = 45\cos(20°)\times 2$

$= \underline{84.57\ m}$

$y = V\sin(\alpha)\, t + \frac{1}{2} a t^2$

$= 45\sin(20)\times 2 + \frac{1}{2}(-9.81)(2)^2$

$= \underline{11.16\ m}$

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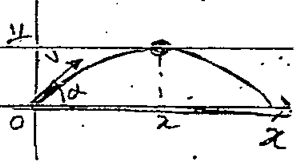
Homework
Repeat Ex(3) & (4) for $V = 40\ m/s$ @ 30°

Ex. 4

What is the highest point reached by the ball and its horizontal distance.

- At the highest point, the (final) vertical velocity is zero.



Using Eqn $a = \frac{(v-u)}{t}$

where $a = -g$
~~or -Weight~~ (acceleration)

$\therefore v = u + at$

we have:

$v = 0$, $u = 45\sin(20°)$ & $a = -9.81$

$\therefore 0 = 45\sin(20°) + (-9.81)t$

$\therefore t = \frac{-45\sin(20°)}{-9.81} = \underline{1.569\ s}$

Calculate the 'x' & 'y' values for $t = \underline{1.569}\ s$