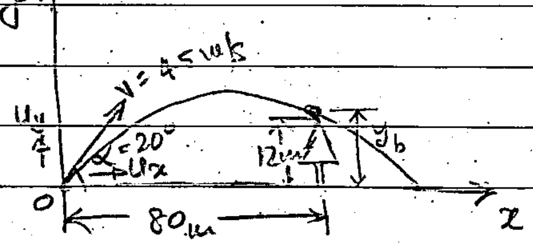
U3A Maths

09-Jun-2026 Term 2 / week 8

Projectile Motion (Cont'd)

Home Work

Will the golf ball clear a 12 m high tree at a dist of 80 m.



We need to find the height of the ball (y_b) at dist $x = 80$ m.

we have, $u_x = V\cos(\alpha) = 45\cos(20°)$

$u_y = V\sin(\alpha) = 45\sin(20°)$

-2-

Horizontal dist. travelled

$$x = u_x . t$$

we have $x = 80$ m

$u_x = 45\cos(20°)$

$$\therefore t = \frac{x}{u_x} = \frac{80}{45\cos(20°)}$$

$$= 1.892 \text{ s}$$

Vertical distance travelled in time 't' is

$$y_b = u_y t + \tfrac{1}{2}at^2$$

where $u_y = 45\sin(20°)$

$t = 1.892$ s

$a = -g = -9.81 \text{ m/s}^2$

$$\therefore y_b = 45\sin(20°) \times 1.892 + \frac{1}{2}(-9.81)(1.892)^2$$

$$= 11.56 \text{ m} \quad (< 12 \text{ m})$$

-3-

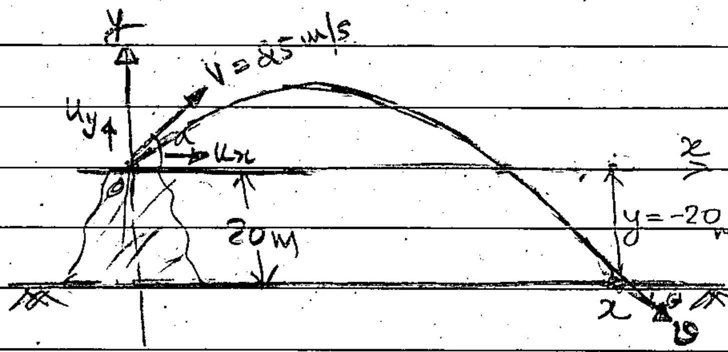
The ball will hit the tree since the tree height = 12 m.

Ex. 1 A rock is ejected from a volcano with a speed of 25 m/s at an angle of 35° to the horizontal. The volcano is at a height of 20 m above the ground level (as shown in the fig.)

(a) calculate the time taken for the rock to reach the ground

(b) what is the magnitude and direction of the rock velocity when it hits the ground.

-4-



We have $u_x = V\cos(\alpha)$

$= 25\cos(35°)$

$u_y = V\sin(\alpha)$

$= 25\sin(35°)$

If we use the surface of the volcano as our reference axes, at the point of impact; $y = -20$ m

we have equations.

$$x = u_x . t \quad -(1)$$

$$y = u_y . t + \tfrac{1}{2}at^2 \quad -(2)$$

where $a = -g = -9.81 \text{ m/s}^2$

Hence, using Equ (2)

$$-20 = 25\sin(35^\circ)t + \frac{1}{2}(-9.81)t^2$$

Rewriting

$$4.905t^2 - 14.34t - 20 = 0$$

We need to solve quadratic Equ.

$$\therefore t = \frac{-(-14.34) \pm \sqrt{(-14.34)^2 - 4(4.905)(-20)}}{2 \times 4.905}$$

$$= \frac{+14.34 \pm \sqrt{598.036}}{9.81}$$

$$= +3.955\,s \quad \text{or} \quad -1.031\,s$$

We can ignore negative value.

Note: What does $-1.031\,s$ represent in practice!?

∴ time taken to hit the ground, $t = 3.955\,s$

(b) For the velocity on impact to the ground, let us assume 'v' as the (final) velocity.

- It has v_x & v_y components.
- No change in horizontal velocity:

$$\therefore v_x = u_x = 25\cos(35^\circ)$$
$$= 20.479\ m/s$$

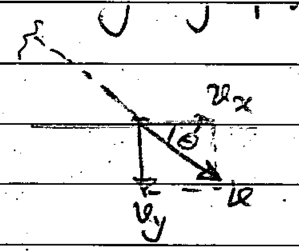
- Final vertical velocity:

$$v_y = u_y + at$$
where $a = -g = -9.81\ m/s^2$

$$\therefore v_y = 25\sin(35^\circ) - 9.81 \times 3.955$$
$$= -24.459\ m/s$$

Note: v_y is negative since our assumed positive direction is rock going up!

∴ Final velocity (magnitude).



$$|v| = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(20.479)^2 + (-24.459)^2}$$

$$= 31.9\ m/s^2$$

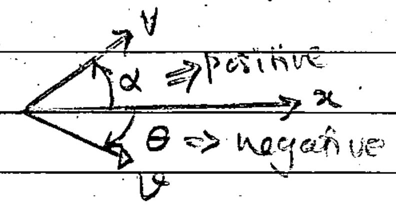
The angle of impact

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

$$\therefore \theta = \tan^{-1}\left(\frac{-24.459}{20.479}\right)$$

$$= -50.06^\circ$$

Note: θ is negative due to our assumed reference.



* Home work *

Ex:3 A fireworks shell is shot into the air at a velocity of 70 m/s at an angle of 75° to the horizontal. The fuse is timed to ignite the shell when it reaches maximum height.

(a) Calculate the height at which shell explodes.

(b) How much time will pass between the launch & the Explosion.

(c) what is the horizontal displacement of the shell when it explodes.

[Ans: (a) 233 m (b) 6.9 s (c) 125 m]

{ Home work }

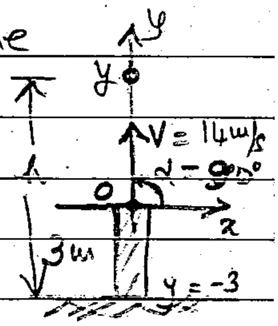
Ex.4

A ball is thrown straight up, from 3m above ground, with a velocity of 14 m/s.

(a) what is the maximum height reached by the ball

(b) At what time does the ball hit the ground.

(c) Plot ball height v/s time



we have

$U_x = V\cos(\alpha)$

$= 14 \times \cos(90°) = \underline{0}$

$U_y = V\sin(\alpha)$

$= 14 \times \sin(90°)$

$= 14$ m/s Also $a = -g = \underline{-9.81}$

we have, initial velocity,

$U_x = V\cos 90° = 14 \times 0 = \underline{0}$

$U_y = V\sin(90°)$

$= 14 \times 1 = \underline{14}$ m/s.

(a) At max. height, final vertical velocity $V_y = 0$;

we have, $V_y = U_y + at$

$0 = 14 + (-9.81)t$

$\therefore t = \frac{-14}{-9.81} = \underline{1.427\,s}$

$\therefore$ Max height, (using our ref. axes)

$y = U_y t + \frac{1}{2}at^2$

$= 14 \times 1.427 + \frac{1}{2}(-9.81)(1.427)^2$

$= \underline{9.99\text{ m}}$

$\therefore$ Height from ground level $= 3 + 9.99$

$= \underline{12.99\text{ m}}$

(b) Using our ref axis, the ground is at $y = -3$ m

we have,

$y = U_x t + \frac{1}{2}at^2$

ie, $-3 = 14t + \frac{1}{2}(-9.81)t^2$

or $4.905t^2 - 14t - 3 = 0$

Solving $t = \underline{3.054\,s}$ or $-0.2\,s$

(c)