U3A Maths

28-Jul-2026 Term3/Week2

Complex Function Plotting

- Consider the complex function (analogous to $f(x) = x^2 - 4$)

$f(\bar{z}) = \bar{z}^2 - 4$

where $\bar{z}$ is a complex variable

- Let us first calculate the roots of the above function

$$f(\bar{z}) = \bar{z}^2 - 4 = 0$$
$$\therefore \bar{z}^2 = 4$$
$$\bar{z} = \pm 2$$
$$= (+2+i0) \;\&\; (-2+i0)$$

$\therefore$ The roots are $\bar{z} = (x+iy) = (2+i0) \;\&\; (-2+i0)$

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- Similarly, we can calculate roots of function $f(\bar{z}) = \bar{z}+4$!
- Let us now plot $f(\bar{z})$

$\bar{z} = (x+iy) \Rightarrow$ the input values

$\bar{w} = f(\bar{z}) = \bar{z}^2 - 4$

let $\bar{w} = (u+iv) \Rightarrow$ the output values

or $\bar{w} = |w| \angle\phi \Rightarrow$ Alternative output values

- We have two input variables, namely, x & y

and two output variables u & v or $|w|$ & ϕ

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- Hence, we need 4 dimensions to plot ! — Not feasible since we have only 3 dimensions
- Note that we can calculate the two output values, that is, $(u+iv)$ or $|w| \angle\phi$, but can only plot one of them.
- Let us choose $|w|$ or magnitude for our plots. Other values can be plotted, if needed.
- Let us now establish a systematic process to obtain the plot inputs & outputs

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- Let us consider the following range for input values

$x \Rightarrow -4$ to $+4$ in steps of 0.25

$y \Rightarrow -4$ to $+4$ in steps of 0.25

- The above ranges can be expressed as an array of values

$$x = \left[\overset{x_1}{-4}, \overset{x_2}{-3.75}, \overset{x_3}{-3.5}, \ldots \overset{x_{17}}{0}, \ldots, \overset{x_{31}}{+3.5}, \overset{x_{32}}{+3.75}, \overset{x_{33}}{+4}\right]$$

$$y = \left[\underset{y_1}{-4}, \underset{y_2}{-3.75}, \underset{y_3}{-3.5}, \ldots \underset{y_{17}}{0}, \ldots, \underset{y_{31}}{+3.5}, \underset{y_{32}}{+3.75}, \underset{y_{33}}{+4}\right]$$

- For calculation purposes, we need to express the the above values in complex form for all combinations, ~~Ex~~ Ex.

$(x_1 + iy_1) = (-4 - i4)$

$(x_1 + iy_2) = (-4 - i3.75)$ etc.

- The above is more conveniently expressed in <u>Matrix</u> form.

	-4	-3.75	-3.5	-3.25	-3.0		+3.75	+4
$y \backslash x$	x_1	x_2	x_3	x_4	x_5	....	x_{32}	x_{33}
-4 y_1	(-4 - i4)	(-3.75 - i4)	(-3.5 - i4)	(-3.25 - i4)	(-3.0 - i4)	....	(3.75 - i4)	(+4 - i4)
-3.75 y_2	(-4 - i3.75)	(-3.75 - i3.75)	(-3.5 - i3.75)	(-3.25 - i3.75)	(-3.0, -i3.75)	....	(3.75 - i3.75)	(4 - i3.75)
-3.5 y_3	(-4 - i3.5)	(-3.75 - i3.5)	(-3.5 - i3.5)	(-3.25 - i3.5)	(-3.0 - i3.5)	....	(3.75 - i3.5)	(4 - i3.5)
-3.25 y_4	(-4 - i3.25)	(-3.75 - i3.25)	(-3.5 - i3.25)	(-3.25 - i3.25)	(-3.0 - i3.25)	....	(3.75 - i3.5)	(4 - i3.5)
-3.0 y_5	(-4 - i3)	(-3.75 - i3)	(-3.5 - i3)	(-3.25 - i3)	(-3.0 - i3)	....	(3.75 - i3)	(4 - i3)
⋮			⋮				⋮	
+3.75 ⋮	(-4 + i3.75)	(-3.75 + i3.75)	(-3.5 + i3.75)	(-3.25 + i3.75)	(-3.0 + i3.75)	....	(3.75 + i3.75)	(4 + i3.75)
+4 y_{33}	(-4 + i4)	(-3.75 + i4)	(-3.5 + i4)	(-3.25 + i4)	(-3.0 + i4)	....	(3.75 + i4)	(4 + i4)

- Hence, there are a total of 33×33 = 1089 complex values ie, $\bar{z} = (x + iy)$
- We need to calculate $f(\bar{z}) = \bar{z}^2 - 4$ for each of the above values
- Hence, $\bar{w} = f(\bar{z})$ will also be a 33 × 33 = 1089 complex values.
- A part of $f(\bar{z})$ matrix of values are given in <u>30-Jun-2026</u> notes!

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- Let us calculate and check the $f(\bar{z})$ for $\bar{z}$ values circled above!

<u>Row (1) - Col. (1)</u>

- $\bar{z}_1 = (x_1 + iy_1) = (-4 - i4)$

$$f(\bar{z}_1) = \bar{z}_1^2 - 4 = [(4) + (-i4)]^2 - 4$$

$$= (-4)^2 + 2(-4)(-i4) + (-i4)^2 - 4$$

$$= \cancel{16} + i32 - \cancel{16} - 4 = \underline{(-4 + i32)}$$

<u>Row (1) - Col. (5)</u>

$$\bar{z} = (x_5 + iy_1) = (-3 - i4)$$

$$f(\bar{z}) = \bar{z}^2 - 4 = [(-3) + (-i4)]^2 - 4$$

$$= (-3)^2 + 2(-3)(-i4) + (-i4)^2 - 4$$

$$= 9 + i24 - 16 - 4 = \underline{(-11 + i24)}$$

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<u>Row (5) - Col (1)</u>

$$\bar{z} = (x_1 + iy_5) = (-4 - i3)$$

$$f(\bar{z}) = \bar{z}^2 - 4 = [(-4) + (-i3)]^2 - 4$$

$$= (-4)^2 + 2(-4)(-i3) + (-i3)^2 - 4$$

$$= 16 + i24 - 9 - 4 = \underline{(3 + i24)}$$

<u>Row (5) - Col (5)</u>

$$\bar{z} = (x_5 + iy_5) = (-3 - i3)$$

$$f(\bar{z}) = \bar{z}^2 - 4 = [(-3) + (-i3)]^2 - 4$$

$$= (-3)^2 + 2(-3)(-i3) + (-i3)^2 - 4$$

$$= \cancel{9} + i18 - \cancel{9} - 4$$

$$= \underline{(-4 + i18)}$$

- Demonstration of various plots using GNU-Octave.
- Refer to Term 2 / Week 11 Notes for plots.