U3A Maths

4-Aug-2026 Term 3 / Week 3

Complex Plots - Domain Coloring

- Last week we plotted complex functions using 3 dimensional plots. However, we could plot one of the parameters of the complex function outputs, namely, Real Part, Imaginary Part, Magnitude or the phase angle.
- The above produced impressive plots and was useful in illustrating real and imaginary roots.

- Another powerful way of plotting is called "Domain Coloring"
- Domain colouring is used to plot both 'magnitude' and 'angle' of the complex function output simultaneously.
- In addition, colour coding is used to make the plots more impressive and powerful.
- These plots are essentially plots on complex (Re, Im) axes.

- Let us consider our example function, namely

$$f(z) = z^2 + 4$$

where z is a complex variable

Ex.1 Calculate $f(z)$ for

$$z = (x+iy) = (1+i1)$$

Let,

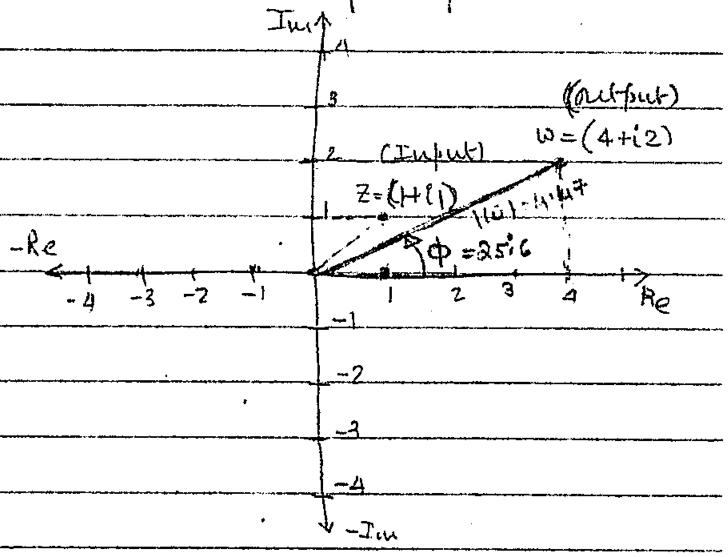
$$w = f(z) = (1+i1)^2 + 4 = 1 + 2i - 1 + 4$$
$$= (4 + i2)$$
$$= \sqrt{(4^2+2^2)} \angle \tan^{-1}(2/4)$$
$$= 4.472 \angle 25^\circ.57$$

Hence, function output values are,

Magnitude ⇒ $|w| = 4.472$

Phase Angle ⇒ $\phi = 26^\circ.57$

- We can plot these values on the complex plane



- We have plotted both input values (z=1+i1) and the function output values (w=4+i2) above.

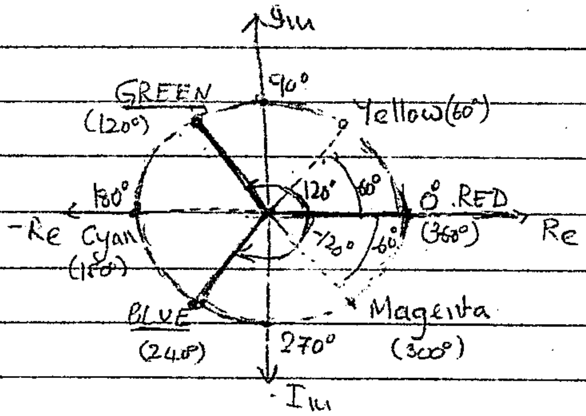
- Since, we are interested only in the function output values, there is really no need to plot the input values, namely, 'z' values.

- For the present, let us say we are only interested in the "Phase Angle" values for plotting!

- In addition, let us assign a colour to the angle, using the Rainbow Colours.
- In computer displays, Rainbow Colours are obtained

by a combination of Red (R), Green (G) and Blue (B) Colours at a given point (pixel) on the screen.

- The colour coding for the 'phase angle' can be done as shown below:



Colour Code:

Angle ⇒	0°	120°	240°	360°
	R	G	B	R

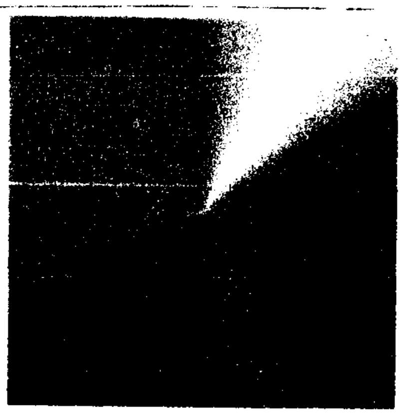
- Colours for other angles are obtained by mixing the R-G-B colours proportionately

For Ex:

60° ⇒ 50% Red + 50% Green = Yellow!

Note: The above mixing only applies to the mixing of 'active' colours, such as light rays! The 'passive' or paint mixing results in different colours.

- For our example, the phase angle of 25.6° will be close to 'orange' colour! Hence, the point 'ω' will be plotted as "orange".
- The range of colours using this scheme is best illustrated by considering $\omega = f(z) = z$, as below



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- Magnitudes can be plotted using the brightness! However, in practice it is common to illustrate the magnitudes by "Contour lines" corresponding to constant magnitude values – just like weather maps and land survey maps.

- Magnitude contour lines are most commonly illustrated on a logarithmic scale. In addition, the brightness is scaled between the contour lines. This helps to illustrate the zeros (Roots) and Infinity (peaks) more clearly.

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- This color scheme is called "HSV".

  H – Hue ⇒ Phase Angle (Colour)

  S – Saturation ⇒ Ignored! (Constant)

  V – Value ⇒ Magnitude (Brightness)

- Magnitude contours for $f(z) = z$ is as shown below:

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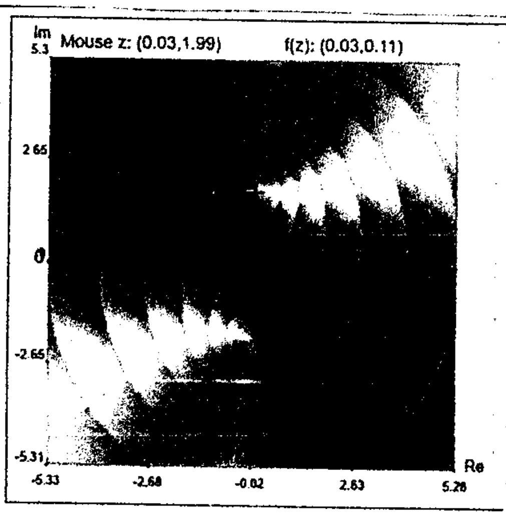
- Finally, we can now plot our example function $w = f(z) = z^2 + 4$. Note the location of the roots on the plot. In the Figure, note that for $z = (0.03 + i1.99)$ $f(z) = (0.03 + i\,0.11)$



Homework: Calculate $f(z)$ for $z = (0.03 + i1.99)$ and check your results!

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- The above plots were obtained interactively from the website "https://complex-analysis.com/content/domain_coloring.html"

- We can try plotting the following functions!

$f(z) \Rightarrow z, z^{-1}, z^2, z^{-2}$, etc.

$f(z) \Rightarrow (z-1)/(z^n+z+1)$ for $n = 2, 3, \ldots$

$f(z) \Rightarrow [(1/z)^n - (1/z)]/(1/z - 1)$ for $n = 1, 3, 5$, etc.

$f(z) \Rightarrow e^{1/z}$

$f(z) \Rightarrow \sin(1/z^2)$

$f(z) \Rightarrow \log(z)$

$f(z) = z^{(2/3+i)}$

$f(z) = z/(1-z) + z^2/(1-z^2) + z^3/(1-z^3) + \cdots$