U3A Maths

1-Sep-2026 Term3 / Week7

Aerofoil & Trans. Line - Examples

- **Ex. 1** (Homework)
  - Air craft wind chord length (circumference of aerofoil) $C = 10\,m$
  - Lift Force Reqd.. $L = 3560$ kN at take off speed $V = 282$ km/h.
  - Lift Coefficient $C_L = 1.0$ (Angle of attack ≙ 10°)
  - Air density at sea level $\rho = 1.225$ Kg/m³

Find the total wing span. (both the wings!)

We have Lift Force Equ.

$$L = C_L \cdot \tfrac{1}{2}\rho V^2 A \quad \text{Newtons}$$

$$V = 282 \text{ km/h} = \frac{282 \times 1000}{(60 \times 60)} \text{ m/s}$$

$$= 78.333 \text{ m/s}$$

$$\therefore A = \frac{3560 \times 10^3}{1.0 \times \frac{1}{2} \times 1.225 \times (78.333)^2}$$

wing Area

$$= 947.23 \text{ m}^2$$

$$\therefore \text{Wing Span (length)} = \frac{A}{C} = \frac{947.23}{10}$$

$$= \underline{94.723 \text{ m}}$$

Note:

Airbus A380 : Wing span = 80 m.
Wing Area = 845 m²

- Newton ($N = kg\,m/s^2$) is not a popularly used unit of Force.
- A more common unit is the Force exerted by 1kg due to gravity. This is called "1 kgf"
- We have, Force = Mass × Accln

$$\therefore 1\text{ N} = 1\text{ kg} \times 1\text{ m/s}^2$$

$$1\text{ kgf} = 1\text{ kg} \times 9.81\text{ m/s}^2$$

(Accln due to gravity)

$$\therefore \boxed{1\text{ kgf} = 9.81\text{ N}}$$

or

$$\boxed{1\text{ N} = \frac{1}{9.81} = 0.1019\text{ kgf}}$$

Note: 1 Kgf is force needed to Lift 1kg against gravity!

Ex. 2 Express the Lift force (L) in Ex. 1 in Kgf

$$L = 3560 \times 10^3 \text{ Newtons}$$

$$= \frac{3560 \times 10^3}{9.81} \text{ kgf}$$

$$= \underline{362.9 \times 10^3} \text{ kgf}$$

The above force is enough to lift $\underline{362.9 \times 10^3}$ kg or $\underline{362.9}$ tonnes

Notes:

A380 Empty weight 277 tonnes
A380 Max. Landing weight 369 tonnes

Ex. 3

What is the lift force required by a Boing 737 jet at a crusing height of 11,000 m. It is given that:

- Lift factor $C_L \cong 0.5$ (level flight) (Angle of attack $\cong 2°$ to $5°$)
- Air density (at 11,000 m) $\rho \cong 0.365$ kg/m^3
- Wing Area $A = 125$ m^2
- Cruise Velocity $V = 230$ m/s (828 km/h)

We have

$$L = C_L \cdot \tfrac{1}{2} \rho V^2 A$$

$$= 0.5 \times \tfrac{1}{2} \times 0.365 \times (230)^2 \times 125$$

$$= \underline{\underline{603{,}390 \text{ N}}} \text{ or } \underline{\underline{61{,}508 \text{ kg-f}}}$$

Ex. 4

What is the wing area required for a wind turbine to produce 5,000 N at 70 m/s wind speed.

Given:

$C_L = 1.1$ (Typical for S-Series Airfoil)

$\rho = 1.225$ kg/m^2

We have $L = C_L \cdot \tfrac{1}{2} \rho V^2 \cdot A$

$$\therefore A = \frac{5000}{1.1 \times \tfrac{1}{2} \times 1.225 \times (70)^2}$$

$$= \underline{1.52 \text{ m}^2}$$

Note: If chord length is 1.5 m then wing span is $\cong$ 1 m.

A.C. Transmission line

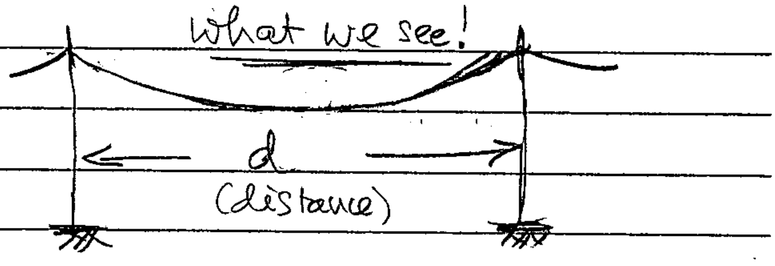
- A typical bare electrical conductor hanging in air has following electrical parameters (approx)

- Resistance (r) $= \underline{0.1}$ ohm/km (100 mm^2 ACSR Conductor)
- Inductive Reactance (x_L) $= \underline{0.5}$ ohm/km (due to Magnetic field) ($\cong$ constant)
- Capacitive Reactance (x_c) $= \underline{300{,}000}$ ohm/km. ($\cong$ constant) (due to electrostatic field) ~~(Air acts like a battery!)~~

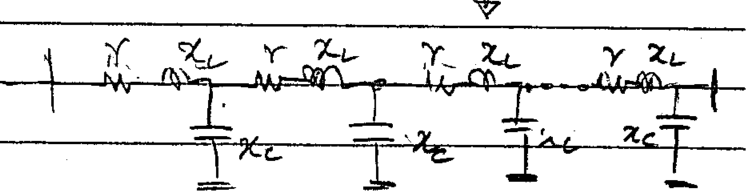
Note: • X_L & X_c values are independent of conductor size (diameter)

• It is marginally affected by (Phase) conductor separation.

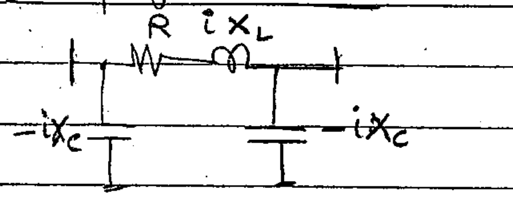
What we see!



What it is!



Simplified Model



Division since x_c's are parallel

$R = r \times d$; $X_L = x_L \times d$; $X_c = \dfrac{x_c}{(d/2)}$

$iX_L \Rightarrow$ Since voltage across inductance 'Leads' the current by $90°$.

$-iX_C \Rightarrow$ Since voltage across capacitance 'Lags' the current by $90°$.

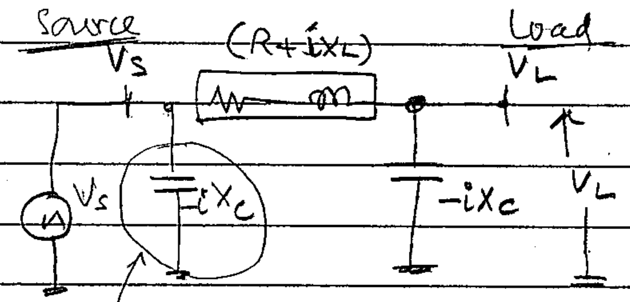
Ex. 1 Calculate the voltage at the load end for a 10km long line connected to a 11KV source. (For simplicity assume single phase line)

Given:

$R = 0.1\,\Omega/km \times 10km = 1\,\Omega$

$X_L = 0.5\,\Omega/km \times 10km = 5\,\Omega$

$X_C = 300{,}000\,\Omega/km \div (10km/2)$
$= 60{,}000\,\Omega$



We can ignore this, since the value of V_S is given $(= 11\,kV)$

Using voltage divider rule:

$$\frac{V_L}{V_S} = \frac{(-iX_C)}{(R+iX_L)+(-iX_C)}.$$

$$\therefore V_L = \frac{-i\,60{,}000}{(1+i5)+(-i\,60{,}000)} \times 11\,kV\angle 0°$$

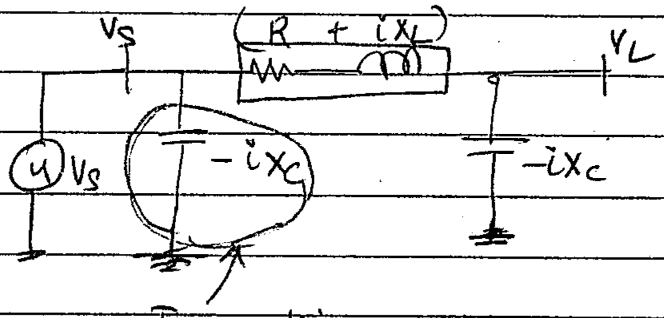
$$\simeq \underline{\underline{11\,kV\angle 0°}}$$

Ex. 2 Repeat the above Example for a 500KV source and a 500 km long line.

$R = 0.1\,\Omega/km \times 500km = 50\,\Omega$

$X_L = 0.5\,\Omega/km \times 500km = 250\,\Omega$

$X_C = 300{,}000\,\Omega/km \div \left(\frac{500km}{2}\right) = 1200\,\Omega$



Ignore this as V_S is given

- Using voltage divider rule, we have

$$V_L = \frac{(-iX_C)\cdot V_S}{(R+iX_L)+(-iX_C)} = \frac{-i\,1200 \times 500\angle 0°}{(50+i250)+(-i1200)}$$

$$= \frac{1200\angle -90°}{(50-i950)} \times 500\angle 0°$$

$$= \frac{1200\angle -90°}{951.3\angle -87°} \times 500\angle 0°$$

$$= 631\,kV\angle -3°$$ (26% higher voltage!)

<u>Demonstration – Complex Function Plot</u>

Iterative plot of complex function

$$f(z) = \frac{\cos(z)}{\sin(z^4-1)}$$ {Persian Carpet!!}

Web site Ref:

1. Jedsoft.org/fun/complex/
2. peterefrancis.com/complex-function-plot/