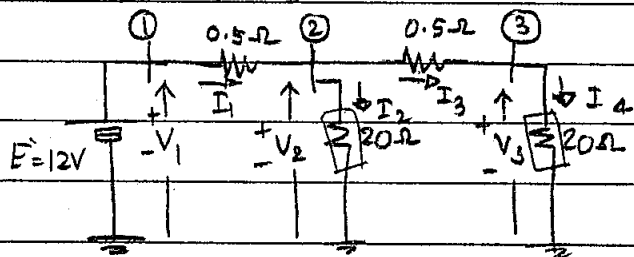


Basic Algebra Practical Examples

Ex.1 (Homework)

Calculate the node voltages

V_2 and V_3 .



Node ① voltage is given, i.e.,
 $V_1 = E = 12V$

- 3 -

KCL & Ohm's Law at Node ③

$$I_3 = I_4$$

$$\frac{(V_2 - V_3)}{0.5} = \frac{V_3}{20} \quad (\times 20)$$

i.e., $40(V_2 - V_3) = V_3$

$$40V_2 - 41V_3 = 0 \quad \text{--- (2)}$$

$$\text{or } V_2 = \frac{41}{40} V_3$$

Substituting Eqn (2) in Eqn (1),

$$81 \left(\frac{41}{40} \right) V_3 - 40V_3 = 480$$

$$\therefore V_3 = \frac{480}{81 \times \frac{41}{40} - 40} = \underline{\underline{11.156V}}$$

Using Eqn (2)

$$V_2 = \frac{41}{40} \times V_3 = \underline{\underline{11.435V}}$$

- 2 -

Using Kirchhoff's Current Law

$$(KCL) \Rightarrow \sum I_{in} = \sum I_{out}$$

(NOTE: $\sum$ represents 'sum')

KCL at Node ②

$$I_1 = I_2 + I_3$$

Using Ohm's law for currents

$$\frac{(V_1 - V_2)}{0.5} = \frac{V_2}{20} + \frac{V_2 - V_3}{0.5} \quad (\times 20)$$

$$40(V_1 - V_2) = V_2 + 40(V_2 - V_3)$$

$$40V_1 - 40V_2 = V_2 + 40V_2 - 40V_3$$

$$\therefore 81V_2 - 40V_3 = 40V_1$$

$$= 40 \times 12 = 480$$

$$\text{i.e., } 81V_2 - 40V_3 = 480 \quad \text{--- (1)}$$

- 4 -

Expressing V_2 & V_3 as % of Nominal.

$$\% V_2 = \frac{11.435}{12} \times 100 = \underline{\underline{95.29\%}}$$

$$\% V_3 = \frac{11.156}{12} \times 100 = \underline{\underline{92.97\%}}$$

Note: The voltage levels are acceptable now!

Let us also calculate the current & power flow in the battery

$$I_1 = \frac{V_1 - V_2}{0.5} = \frac{12 - 11.435}{0.5}$$

$$= \underline{\underline{1.13A}}$$

$$P_1 = V_1 I_1 = 12 \times 1.13$$

$$= \underline{\underline{13.56W}}$$

• Calculate "Power loss" in the cables

we have:

$$\begin{aligned} \text{(Load power flow)} \quad P_2 &= V_2 \cdot I_2 = V_2 \cdot \left(\frac{V_2}{20}\right) \\ &= \frac{(11.435)^2}{20} = \underline{6.54 \text{ W}} \end{aligned}$$

$$\begin{aligned} \text{(Load power flow)} \quad P_3 &= V_3 I_4 = V_3 \left(\frac{V_3}{20}\right) \\ &= \frac{(11.156)^2}{20} = \underline{6.22 \text{ W}} \end{aligned}$$

$$\begin{aligned} \therefore \text{Power loss in cables} &= P_1 - P_2 - P_3 \\ &= 13.56 - 6.54 - 6.22 \\ &= \underline{0.8 \text{ W}} \end{aligned}$$

• Efficiency of transmission:

$$= \frac{\text{Loss}}{\text{Generation}} \times 100 = \frac{0.8}{13.56} \times 100 = \underline{5.9\%}$$

$$\therefore \frac{(V_2 - V_3)}{0.5} = \frac{V_3}{20}$$

$$\frac{(12 - V_3)}{0.5} = \frac{V_3}{20}$$

$$20(12 - V_3) = 0.5 V_3$$

$$\therefore V_3 = \frac{20 \times 12}{(20 + 0.5)} = \underline{11.707 \text{ V}}$$

• Power flow from the battery at node # (2).

$$P_2 = V_2 I_2 + V_2 I_3$$

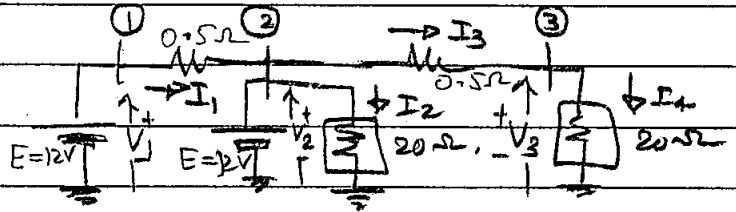
$$= V_2 \left(\frac{V_2}{20}\right) + V_2 \left(\frac{V_2 - V_3}{0.5}\right)$$

$$= \frac{12 \times 12}{20} + \frac{12(12 - 11.707)}{0.5}$$

$$= 7.2 + 7.032 = \underline{14.232 \text{ W}}$$

Ex. 2

Calculate voltages and power flow through the batteries for the following circuit.



No need to solve for V_1 & V_2 as they are known!

$$V_1 = E = 12 \text{ V}$$

$$V_2 = E = 12$$

Using KCL at node (3)

$$I_3 = I_4$$

• Power flow from the battery at node # (1)

$$\begin{aligned} P_1 &= V_1 I_1 \\ &= V_1 \left(\frac{V_1 - V_2}{0.5}\right) \end{aligned}$$

$$= 12 \times \frac{(12 - 12)}{0.5}$$

$$= \underline{0 \text{ W}}$$

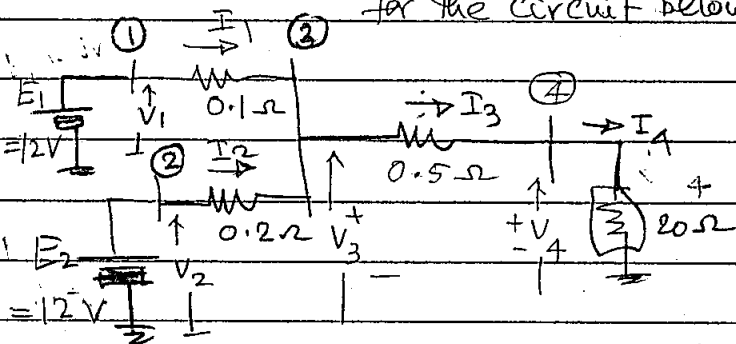
• Battery at node # (1) is not supplying any power!

• This is true if both the batteries are "ideal" sources.

• However, in practice all batteries have internal resistance.

Ex. 3

Calculate the node voltages and battery power flows considering internal resistances for the circuit below.



V_1 & V_2 values are given.

$$V_1 = E_1 = 12V$$

$$V_2 = E_2 = 12V$$

Using KCL at node (3)

$$I_1 + I_2 = I_3$$

Substituting in Eqn (1)

$$8.5 \times \left(\frac{20.5}{20} \right) V_4 - V_4 = 90$$

$$8.7125 V_4 - V_4 = 90$$

$$\therefore V_4 = \frac{90}{(8.7125 - 1)} = \underline{\underline{11.669V}}$$

$$\therefore V_3 = \frac{20.5}{20} \times V_4 = \underline{\underline{11.961V}}$$

Power flow in the battery at node # (1)

$$P_1 = V_1 I_1 = V_1 \frac{(V_1 - V_3)}{0.1}$$

$$= \frac{12(12 - 11.961)}{0.1} = \underline{\underline{4.68W}}$$

Power flow in the battery at node # (2)

$$P_2 = V_2 I_2 = V_2 \frac{(V_2 - V_3)}{0.2}$$

$$= \frac{12(12 - 11.961)}{0.2} = \underline{\underline{2.34W}}$$

$$\frac{(V_1 - V_3)}{0.1} + \frac{(V_2 - V_3)}{0.2} = \frac{(V_3 - V_4)}{0.5}$$

Multiplying both sides by 0.5

$$5(V_1 - V_3) + 2.5(V_2 - V_3) = (V_3 - V_4)$$

$$5(12 - V_3) + 2.5(12 - V_3) = (V_3 - V_4)$$

Rearranging we get,

$$8.5 V_3 - V_4 = 90 \quad \text{--- (1)}$$

Using KCL at node # (4)

$$I_3 = I_4$$

$$\frac{(V_3 - V_4)}{0.5} = \frac{V_4}{20}$$

Rearranging

$$V_3 = \left(\frac{20.5}{20} \right) V_4 \quad \text{--- (2)}$$

• Note that the power sharing is inversely proportional to the internal resistance!

Homework

Solve the network for voltages and power flows

