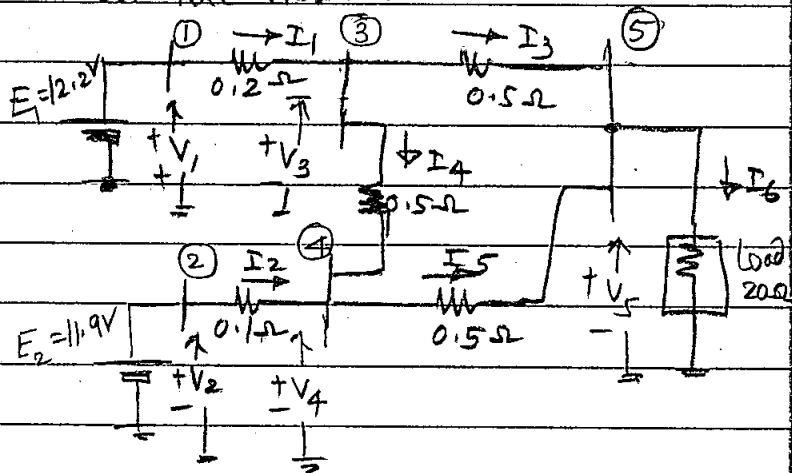


Simultaneous Equations

Ex-1 (Homework Problem)

Solve for the network voltages using Kirchhoff's Current Law (KCL) at the nodes.



- 3 -

Using KCL at (4)

$$I_2 + I_4 = I_5$$

$$\frac{(V_2 - V_4)}{0.1} + \frac{V_3 - V_4}{0.5} = \frac{V_4 - V_5}{0.5}$$

$$\text{i.e., } 10(11.9 - V_4) + 2(V_3 - V_4) = 2(V_4 - V_5)$$

Collecting the terms, we get

$$\underline{-2V_3 + 14V_4 - 2V_5 = 119} \quad \text{--- (2)}$$

Using KCL at (5)

$$I_3 + I_5 = I_6$$

$$\frac{(V_3 - V_5)}{0.5} + \frac{(V_4 - V_5)}{0.5} = \frac{V_5}{20}$$

$$\text{i.e., } 2(V_3 - V_5) + 2(V_4 - V_5) = 0.05V_5$$

Rearranging,

$$\underline{-2V_3 - 2V_4 + 4.05V_5 = 0} \quad \text{--- (3)}$$

- 2 -

No need to solve for V_1 & V_2 as they are known.
We have,

$$V_1 = E_1 = 12.2V$$

$$V_2 = E_2 = 11.9V$$

Using KCL at node (3).

$$I_1 = I_3 + I_4$$

$$\frac{V_1 - V_3}{0.2} = \frac{V_3 - V_5}{0.5} + \frac{V_3 - V_4}{0.5}$$

$$\text{i.e., } 5(12.2 - V_3) = 2(V_3 - V_5) + 2(V_3 - V_4)$$

Collecting the terms, we get

$$\underline{9V_3 - 2V_4 - 2V_5 = 61} \quad \text{--- (1)}$$

- 4 -

• We now have 3 equations with 3 variables.

• Hence, we have a unique solution.

• The equations are:

$$9V_3 - 2V_4 - 2V_5 = 61 \quad \text{--- (1)}$$

$$-2V_3 + 14V_4 - 2V_5 = 119 \quad \text{--- (2)}$$

$$-2V_3 - 2V_4 + 4.05V_5 = 0 \quad \text{--- (3)}$$

• We need to solve the above equations to obtain the values of voltages V_3 , V_4 and V_5

• The simple substitution method used for 2 eqns and 2 variables is difficult to apply in this case.

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Using Octave software, we get,

$$V_3 = 12.055V$$

$$V_4 = 11.913V$$

$$V_5 = 11.836V$$

Ex. 2

Calculate the power flow from batteries at nodes (1) & (2) and the power flow into the load at (5) and the power loss in the network.

Power flow from battery at node (1)

$$P_1 = V_1 I_1 = V_1 \times \frac{(V_1 - V_2)}{0.2}$$

$$= \frac{12.2(12.2 - 12.055)}{0.2} = \underline{\underline{8.845W}}$$

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Power loss in the network:

$$\begin{aligned} P_{\text{Loss}} &= P_1 + P_2 - P_{\text{Load}} \\ &= 8.845 + (-1.547) - 7.005 \\ &= \underline{\underline{0.293 \text{ Watts}}} \end{aligned}$$

How do we know that the results are correct?

& check

Ex. 3 Calculate the power loss in the network resistances, given that the power loss in the resistor $P_R = I^2 \cdot R$.

∴ we need to calculate all the currents in the network.

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Power flow battery at node (2)

$$P_2 = V_2 I_2 = V_2 \cdot \frac{(V_2 - V_4)}{0.1}$$

$$= \frac{11.9(11.9 - 11.913)}{0.1}$$

$$= \underline{\underline{-1.547W}} \quad !!$$

• The value is negative! The battery is actually charging and not supplying load!

Power flow into the load at (5)

$$\begin{aligned} P_5 &= V_5 I_5 \\ &= V_5 \left(\frac{V_5}{20} \right) \end{aligned}$$

$$\begin{aligned} &= \frac{11.836(11.836)}{20} \\ &= \underline{\underline{7.005W}} \end{aligned}$$

-12-

$$I_1 = \frac{V_1 - V_3}{0.2} = \frac{12.2 - 12.055}{0.2} = \underline{\underline{0.725A}}$$

$$I_2 = \frac{V_2 - V_4}{0.1} = \frac{11.9 - 11.913}{0.1} = \underline{\underline{-0.13A}}$$

Current flow is opposite to the assumed direction!

$$I_3 = \frac{V_3 - V_5}{0.5} = \frac{12.055 - 11.836}{0.5} = \underline{\underline{0.438A}}$$

$$I_4 = \frac{V_3 - V_4}{0.5} = \frac{12.055 - 11.913}{0.5} = \underline{\underline{0.284A}}$$

$$I_5 = \frac{V_4 - V_5}{0.5} = \frac{11.913 - 11.836}{0.5} = \underline{\underline{0.274A}}$$

$$\begin{aligned} \therefore \text{Loss in Network resistances} &= P_{\text{Loss}} \\ &= I_1^2 \times 0.2 + I_2^2 \times 0.1 + I_3^2 \times 0.5 + I_4^2 \times 0.5 \\ &\quad + I_5^2 \times 0.5 \end{aligned}$$

$$= 0.1051 + 0.0017 + 0.096 + 0.0603 + 0.0375$$

$$= \underline{\underline{0.2806W}} \quad \left\{ \begin{array}{l} \text{Discrepancy is due to rounding off errors} \end{array} \right.$$

Octave Commands to solve Matrix Form Simultaneous Equations

```
octave:16> A = [9,-2,-2; -2,14,-2; -2,-2,4.05]  
A =
```

```
    9.0000   -2.0000   -2.0000  
   -2.0000   14.0000   -2.0000  
   -2.0000   -2.0000    4.0500
```

```
octave:17> b = [61; 119; 0]  
b =
```

```
    61  
   119  
     0
```

```
octave:18> x = inv(A) * b  
x =
```

```
   12.055  
   11.913  
   11.836
```

```
octave:19>
```