U3A Maths

3-Nov-2026 Term 4 / week 4

Simultaneous Equations in Matrix Form

- One of the main uses of matrix form is the solution of large simultaneous equations.
- Algebraically, the matrix form equation provides elegant and simple equations

Equation Form

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

can be expressed in matrix form as below.

Matrix form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(Rows ①, ②, ③ ↓ ; Columns ① ② ③ →)

- It is important to appreciate that both forms express the "same information" in two different formats.
- Matrix form can be converted into Equation form (and vice versa), if required, using row/col multiplications.
- However, the matrix form provides a simpler way to express the information.

- We can write the Matrix form equation "cryptic" form as below:-

$$[A]\ \bar{x} = \bar{b}$$

$[A]$	$\bar{x}$	$\bar{b}$
Square Matrix ('n' rows x 'n' cols)	Column Matrix ('n' rows x 1 col)	Column Matrix ('n' rows x 1 col.)

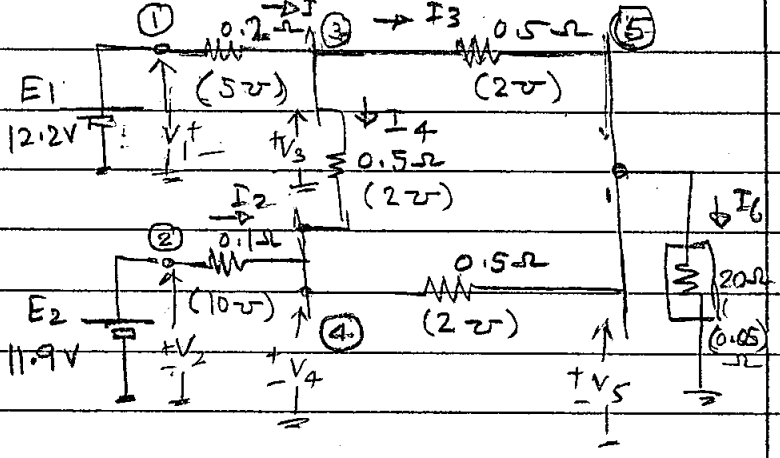
- The above form can be used to represent 'n' equations with 'n' variables (unknowns), where $n = 1, 2, 3, \ldots$
- The solution to the above general equation can be written as,

$$\bar{x} = [A]^{-1}\,\bar{b}$$

- If $[A]^{-1}$ is known, then $\bar{x}$ or $(x_1, x_2, x_3 \ldots)$ can be calculated using simple matrix multiplication.
- In general, the matrix form equations assume that a calculating tool, such a computer is available.
- Quite often, in practical systems, the matrix form equations can be established directly in a simpler way, without recourse to the Equation form!

• Let us redo the last week's problem, without recourse to Equation form.

Ex.1 Solve the network by establishing the matrix directly from the network.



Steps to form the Matrix

1. Convert all resistances to the reciprocal values

For Ex: 0.2 ohms = 5 (mhos)!

Note: 'Mho' is purely a mathematical quantity, but quite often electrical engineers treat it as a 'real' quantity!

2. The diagonal terms of the matrix correspond to summation of the "reciprocal of the resistances" (mhos), connected to that node.

For Ex, at Node ③, the diagonal term is

$5 + 2 + 2 = 9$ ℧

3. The off-diagonal terms are the negative of the "reciprocal of the resistance" connected between the two corresponding nodes.

For Ex: diagonal term corresponding to nodes ③ & ④ is -2 ℧

Note:

We ignore nodes ① & ② as the voltages at these nodes are known.

• We can now form the matrix by inspection of the network.

	③	④	⑤
③	5+2+2	−2	−2
④	−2	10+2+2	−2
⑤	−2	−2	2+2+0.05

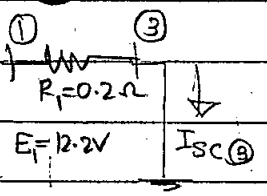
i.e.,

$$[A] = \begin{array}{c} \\ ③ \\ ④ \\ ⑤ \end{array} \begin{array}{c} \begin{array}{ccc} ③ & ④ & ⑤ \end{array} \\ \begin{bmatrix} 9 & -2 & -2 \\ -2 & 14 & -2 \\ -2 & -2 & 4.05 \end{bmatrix} \end{array}$$

[You can cross check with last week's notes!]

- The right hand side (RHS) column matrix ($\bar{b}$) is formed as follows. For each node connected directly to the battery, the value corresponds to the maximum (short circuit) current that can be supplied by the battery.

For Ex., at Node #(3)



$$I_{sc③} = \frac{E_1}{R_1} = \frac{12.2\,V}{0.2\,\Omega} = 61\,A$$

Similarly,

- At node #④, $I_{sc④} = \frac{E_2}{R_2} = \frac{11.9}{0.1} = 119\,A$.

At node #⑤

$I_{sc⑤} = 0$ as there is no battery at node ⑤

We have,

$$\bar{b} = \begin{matrix} ③ \\ ④ \\ ⑤ \end{matrix}\begin{bmatrix} I_{sc③} \\ I_{sc④} \\ I_{sc⑤} \end{bmatrix} = \begin{bmatrix} 61 \\ 119 \\ 0 \end{bmatrix}$$

- Note that R_1 & R_2 are the internal resistances of batteries. Hence, nodes ① & ② are internal to the batteries and not accessible. The external battery terminals correspond to nodes ③ & ④.
- We now have the matrix form Eqn $[A]\bar{x} = \bar{b}$

$$\begin{bmatrix} 9 & -2 & -2 \\ -2 & 14 & -2 \\ -2 & -2 & 4.05 \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 61 \\ 119 \\ 0 \end{bmatrix}$$

where

$$[A] = \begin{bmatrix} 9 & -2 & -2 \\ -2 & 14 & -2 \\ -2 & -2 & 4.05 \end{bmatrix}.$$

and $\bar{b} = \begin{bmatrix} 61 \\ 119 \\ 0 \end{bmatrix}$

- Let us solve using the matrix calculator available in "maths is fun.com" web site. on your phone / tablet.
- Enter "matrix calculator" in the search box (2) of the web site.
- Set the matrix [A] size to 3x3 (default) & set the matrix [B] size to 3x1 using arrow keys.

Home work
solve the network using $E_1 = E_2 = 12V$

- Enter the [A] matrix values in the "yellow box" at the bottom. The row values can be separated by spaces or commas. Each row is ended with "carriage return" <↵> or <Enter>
- After entering the matrix values, click on [to A] button. The values will appear on [A] matrix
- Clear the 'yellow box' and enter $\bar{b}$ values and click on [to B].
- Click on [inv (A)] button to get $[A]^{-1}$. The inverted matrix values appear in the "yellow box". Click on [to A]
- Finally, click on [A·B] button to see the results in the "yellow box"