U3A Maths

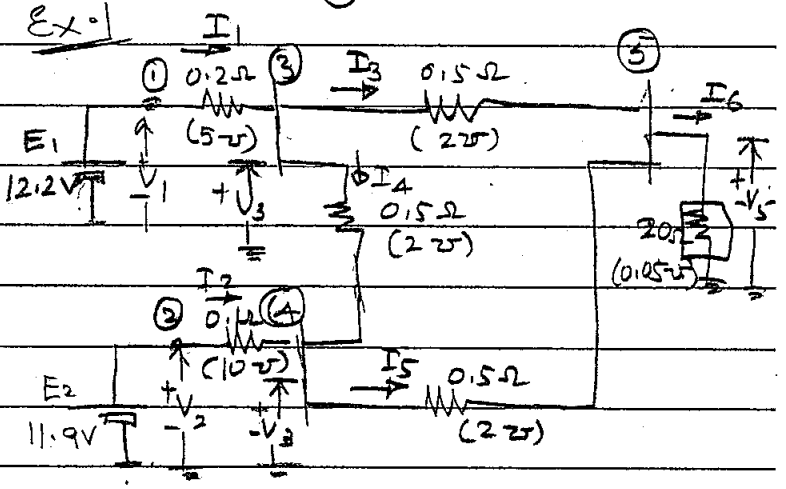
10-Nov-26 Term 4 / Week 5

Matrices in Practice

- Last week we established the matrix (equation) directly from the electric network, using an algorithm.
- Such a matrix can be called network conductance (reciprocal of resistance) matrix.
- However, it is more commonly referred to as "Admittance matrix", since practical networks are A.C. networks and use complex numbers.

- Even though we derived the "Admittance" matrix using an algorithm, the algorithm is essentially derived using Kirchoff's current law Eqns.
- Considering our previous example:

Ex. 1



We can ignore nodes ① & ② since $V_1 = E_1$ & $V_2 = E_2$ are given.

KCL @ node ③

$$I_1 = I_3 + I_4$$

$$\frac{(E_1 - V_3)}{0.2} = \frac{(V_3 - V_5)}{0.5} + \frac{(V_3 - V_4)}{0.5}$$

i.e.,

$$5(E_1 - V_3) = 2(V_3 - V_5) + 2(V_3 - V_4)$$

$$\therefore 5E_1 = 5V_3 + 2V_3 - 2V_5 + 2V_3 - 2V_4$$

$$\therefore \quad 9V_3 - 2V_4 - 2V_5 = 5E_1 \quad -(1)$$

KCL @ node ④ $\quad I_2 + I_4 = I_5$

$$\frac{(E_2 - V_4)}{0.1} + \frac{(V_3 - V_4)}{0.5} = \frac{(V_4 - V_5)}{0.5}$$

i.e.,

$$10(E_2 - V_4) + 2(V_3 - V_4) = 2(V_4 - V_5)$$

$$10E_2 - 10V_4 + 2V_3 - 2V_4 = 2V_4 - 2V_5$$

i.e., $\quad 10E_2 = 10V_4 - 2V_3 + 2V_4 + 2V_4 - 2V_5$

$$= -2V_3 + 14V_4 - 2V_5$$

$$\therefore \quad -2V_3 + 14V_4 - 2V_5 = 10E_2 \quad -(2)$$

KCL at node ⑤

$$I_3 + I_5 = I_6$$

$$\frac{(V_3 - V_5)}{0.5} + \frac{(V_4 - V_5)}{0.5} = \frac{V_5}{20}$$

$$2(V_3 - V_5) + 2(V_4 - V_5) = 0.05V_5$$

i.e., $\quad 2V_3 - 2V_5 + 2V_4 - 2V_5 - 0.05V_5 = 0$

$$2V_3 + 2V_4 - 4.05V_5 = 0$$

or $\quad -2V_3 - 2V_4 + 4.05V_5 = 0 \quad -(3)$

Expressing the above equations in matrix form:

$$\begin{bmatrix} 9 & -2 & -2 \\ -2 & 14 & -2 \\ -2 & -2 & 4.05 \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 5E_1 \\ 10E_2 \\ 0 \end{bmatrix}$$

$$[A]\ \bar{x} = \bar{b}$$

- We have the same matrix, as obtained by Algorithm Method.
- Right hand column is battery volt (E) / battery Resistance
- It is interesting to note that the "Admittance" matrix remains unchanged, when the battery voltages (E_1 & E_2) are changed.
- Hence, matrix solution becomes more efficient when only the battery voltages are changed.

- Let us solve the network again..
- Let us call $[A]^{-1}$ as matrix $[Z]$

$$[Z] = [A]^{-1} = \begin{bmatrix} 0.1365 & 0.0313 & 0.0829 \\ 0.0313 & 0.0841 & 0.0570 \\ 0.0829 & 0.0570 & 0.3160 \end{bmatrix}$$

Notes (using "Mathsisfun.com" Matrix calculator)

($[A]^{-1}$ matrix values are 4 digit precision)

- We have, $E_1 = 12.2V$ & $E_2 = 11.9V$,

$$\therefore \quad \bar{b} = \begin{bmatrix} 5E_1 \\ 10E_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 61 \\ 119 \\ 0 \end{bmatrix}$$

- Using $\bar{x} = [A]^{-1} \times \bar{b} = [Z]\,\bar{b}$

$$\begin{bmatrix} V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0.1365 & 0.0313 & 0.0829 \\ 0.0313 & 0.0841 & 0.0570 \\ 0.0829 & 0.0570 & 0.3160 \end{bmatrix} \begin{bmatrix} 61 \\ 119 \\ 0 \end{bmatrix}$$

- Using matrix multiplication

$V_3 = 0.1365 \times 61 + 0.0813 \times 119 + 0.0829 \times 0$

$= \underline{12.0512\,V}$

$V_4 = 0.0313 \times 61 + 0.0841 \times 119 + 0.0570 \times 0$

$= \underline{11.9172\,V}$

$V_5 = 0.0829 \times 61 + 0.0570 \times 119 + 0.3160 \times 0$

$= \underline{11.8399\,V}$

Notes: - $V_3 = 12.0551V$, $V_4 = 11.913V$, $V_5 = 11.836V$ when using a computer (18 digit precision)

- In practice we need to use at least 5 to 6 digit precision for obtaining acceptable voltage values!

- Let us calculate the battery currents,

$$I_1 = \frac{V_1 - V_3}{0.2} = \frac{(12.2 - 12.055)}{0.2} = \underline{0.725A}$$

$$I_2 = \frac{V_2 - V_4}{0.1} = \frac{(11.9 - 11.913)}{0.1} = \underline{-0.13A}$$

Battery E_1 is supplying the load and Battery E_2 is charging. (I_2 is negative!)

Ex.2

Calculate the battery currents given that $E_1 = 12V$ & $E_2 = 12$

Since, only the battery voltages have changed, the network ("Admittance") matrix does not change. Hence, we have,

$[A]\,\bar{x} = \bar{b}$ as below:

$$\begin{bmatrix} 9 & -2 & -2 \\ -2 & 14 & -2 \\ -2 & -2 & 4.05 \end{bmatrix} \begin{bmatrix} V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 5E_1 \\ 10E_2 \\ 0 \end{bmatrix}$$

Does not change! This changes

- We now have $E_1 = 12V$ & $E_2 = 12V$:

$$\therefore \bar{b} = \begin{bmatrix} 5\times12 \\ 10\times12 \\ 0 \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ 0 \end{bmatrix}$$

∴ Voltages can be calculated using $[Z]$ matrix which has not changed.

i.e. $\bar{x} = [Z]\,\bar{b}$

- Let use 6 digit precision for $[Z]$, ∴ we have,

$$\begin{bmatrix} V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 0.136493 & 0.031339 & 0.082880 \\ 0.031339 & 0.084046 & 0.056980 \\ 0.082880 & 0.056980 & 0.315980 \end{bmatrix} \begin{bmatrix} 60 \\ 120 \\ 0 \end{bmatrix}$$

$\therefore V_3 = 0.136493\times60 + 0.031339\times120 + 0$

$= \underline{11.950\,V}$

$V_4 = 0.031339\times60 + 0.084046\times120 + 0$

$= \underline{11.967\,V}$

$V_5 = 0.082880\times60 + 0.056980\times120 + 0$

$= \underline{11.810\,V}$

- Voltage values when using a computer (with 18 digit precision)

	(18 dig prec)	(6 digit prec.)
V_3 =	11.950 V	11.950V
V_4 =	11.966 V	11.967V
V_5 =	11.810 V	11.810V

- Note that the voltages are acceptable when 6 digit precision is used.

- Calculate new battery currents,

$$I_1 = \frac{V_1 - V_3}{0.2} = \frac{(12 - 11.95)}{0.2} = \underline{0.25A}$$

$$I_2 = \frac{V_2 - V_4}{0.1} = \frac{(12 - 11.966)}{0.1} = \underline{0.34A}$$

- Both batteries E_1 & E_2 are supplying the load! ($I_1 = 0.25A$ & $I_2 = 0.34A$)

- Battery E_2 is supplying higher current, as it has a lower internal resistance. That is, 0.1 Ω compared to 0.2 Ω of battery E_1.